Ex. Solve the following system of equations.

$x+2y+z=7, \quad x+3z=11; \quad 2x-3y=1$

Sol^n. The given system of equation in the form

$$AX=B \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

Let $A=\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$ and the Augmented matrix ~~so~~ $C=[A,B]$ ~~By reducing~~

So, $C=\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 0 & 3 & 11 \\ 2 & -3 & 0 & 1 \end{array}\right]$ By row reducing R_2-R_1, R_3-2R_1

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -2 & 2 & 4 \\ 0 & -7 & -2 & -13 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & -7 & -2 & -13 \end{array}\right]$$

by $R_3 - 7R_2$ $\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & -9 & -27 \end{array}\right]$ i.e. $\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ 2 \\ -27 \end{bmatrix}$

Now it ~~can be~~ is in the form

$$x+2y+z=7 \quad \text{— (1)}$$
$$-y+z=2 \quad \text{— (2)}$$
$$-9z=-27 \quad \text{— (3)} \Rightarrow z=3$$

by back substitution from (2) $\quad -y+3=2 \Rightarrow -y=-1 \Rightarrow y=1$

Now from (1) $\quad x+2\times1+3=7 \Rightarrow x=7-5=2$

hence the required sol^n is $\quad x=2, \; y=1, \; z=3$

Ex Solve the following equations.

$x-y+2z=3$;
$x+2y+3z=5$
$3x-4y-5z=-13$

Solution: The equation are written in Matrix form.

$$\begin{bmatrix}1&-1&2\\1&2&3\\3&-4&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\5\\-13\end{bmatrix}$$

Augmented Matrix is $\begin{bmatrix}1&-1&2&:&3\\1&2&3&:&5\\3&-4&-5&:&-13\end{bmatrix}$

by R_2-R_1, R_3-3R_1 $\sim\begin{bmatrix}1&-1&2&:&3\\0&3&1&:&2\\0&-1&-11&:&-22\end{bmatrix}$

By $R_3+\frac{1}{3}R_2$ $\sim\begin{bmatrix}1&-1&2&:&3\\0&3&1&:&2\\0&0&-\frac{32}{3}&:&-\frac{64}{3}\end{bmatrix}$

Hence $\begin{bmatrix}1&-1&2\\0&3&1\\0&0&-\frac{32}{3}\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\2\\-\frac{64}{3}\end{bmatrix}$

then $-\frac{32z}{3}=-\frac{64}{3} \Rightarrow z=2$ and $3y+z=2$
or $3y+2=2 \Rightarrow y=0$

Now $x-y+2z=3 \Rightarrow x-0+4=3 \Rightarrow x=-1$

Solutions are $x=-1$, $y=0$, $z=2$

Ex. Find the solution of equations by row reducing form:

$x+2y-z=1$
$3x-2y+2z=2$
$7x-2y+3z=5$

or

$x_1+2x_2-x_3=1$
$3x_1-2x_2+2x_3=2$
$7x_1-2x_2+3x_3=5$

Do yourself by row reducing echelon form.

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Paper – GE 2T (Algebra)

linear independence

~~Vectors $x_1, x_2 \ldots x_n$ are said to be dependent if~~

A set of vectors $a_1, a_2, \ldots a_n$ of E^n (in n tupple) is said to be linearly dependent if there exists scalars $\lambda_1, \lambda_2 \ldots \lambda_k$ not all zero such that $\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n = 0$.

If the above equality holds for all $\lambda_i = 0$ $(i = 1, 2, \ldots k)$ then the set of vectors $a_1, a_2 \ldots a_k$ is said to be linearly independent.

or Any vector is expressed as a linear combination of other vectors then the vector set is linearly dependent

i.e if $a_1 = \lambda_2 a_2 + \lambda_3 a_3 + \ldots$ then the vectors are L.D. (linearly dependent)

EX ① Show that the set of vectors $(1, -1, 1, 2)$, $(1, 2, -1, 4)$ and $(1, 1, -1, 5)$ are L.I. (linearly independent.

Sol^n. Let $a_1 = (1, -1, 1, 2)$, $a_2 = (1, 2, -1, 4)$
$a_3 = (1, 1, -1, -5)$ are there exists scalars $\lambda_1, \lambda_2, \lambda_3$ s.t. $\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 = 0$

so $\lambda_1 (1, -1, 1, 2) + \lambda_2 (1, 2, -1, 4) + \lambda_3 (1, 1, -1, -5) = 0$

i.e $\lambda_1 + \lambda_2 + \lambda_3 = 0$ ... ①
$-\lambda_1 + 2\lambda_2 + \lambda_3 = 0$ ... ② $\lambda_1 - \lambda_2 - \lambda_3 = 0$ ... ③
$2\lambda_1 + 4\lambda_2 - 5\lambda_3 = 0$ ... ④

adding ① & ③ $\Rightarrow 2\lambda_1 = 0 \Rightarrow \lambda_1 = 0$
adding ② & ③ $\Rightarrow \lambda_2 = 0$ Putting $\lambda_1 = 0, \lambda_2 = 0$ in ①
we get $\lambda_3 = 0$. which satisfy Equ^n ④
i.e $\lambda_1 = \lambda_2 = \lambda_3 = 0$ i.e all $\lambda_i = 0$. Hence the given set of vectors are linearly independent (L.I.)

Ex. Examine the set of vectors $\{(2,1,1), (1,2,2), (1,1,1)\}$ L.I or L.D. (linearly independent or Linearly dependent.

Sol^n: Let $\lambda_1, \lambda_2, \lambda_3$ be 3 scalar s.t.

$$\lambda_1(2,1,1) + \lambda_2(1,2,2) + \lambda_3(1,1,1) = (0,0,0)$$

$$\Rightarrow \quad 2\lambda_1 + \lambda_2 + \lambda_3 = 0 \quad \text{---} \quad (1)$$

$$\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \quad \text{---} \quad (2)$$

$$\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \quad \text{--} \quad (3)$$

from (1) & (2) $\frac{\lambda_1}{1-2} = \frac{\lambda_2}{1-2} = \frac{\lambda_3}{4-1} = k$ say real no.

$\lambda_1 = -k$, $\lambda_2 = -k$, $\lambda_3 = 3k$. which satisfy (3). As k is real no. arbitrary so $\lambda_1, \lambda_2, \lambda_3$ not all zero. so the given vector are L.D. (linearly Dependent.

Ex. Prove That The vectors $\{(1,2,2), (2,1,2), (2,2,1)\}$ is linearly independent in R^3.

Sol^n: Let c_1, c_2, c_3 be 3 scalar ~~s.t.~~ (real numbers)

s.t. $c_1(1,2,2) + c_2(2,1,2) + c_3(2,2,1) = (0,0,0)$

$$\Rightarrow c_1 + 2c_2 + 2c_3 = 0 \quad \text{———} \quad (1)$$

$$2c_1 + c_2 + 2c_3 = 0 \quad \text{———} \quad (2)$$

$$2c_1 + 2c_2 + c_3 = 0 \quad \text{———} \quad (3)$$

from (1) & (2) $\frac{c_1}{4-2} = \frac{c_2}{4-2} = \frac{c_3}{1-4} = k$ say

Hence $c_1 = 2k$, $c_2 = 2k$ & $c_3 = -3k$ Putting in (3)

$4k + 4k - 3k \neq 0$. possible when $k = 0$

which implies $c_1 = c_2 = c_3 = 0$ (all $c_i = 0$)

This proves That Three vectors are L.I. (linearly independent)