COMPLEX NUMBERS

The system of real numbers is not sufficient to solve all the algebric equ^{n}s. There are no real numbers which satisfy the equ^n $x^2+1=0$ or $x^2=-1$. In order to solve such equ^{n}s, we extend the system of numbers and and introduce a new class of numbers known as imaginary or complex number.

We introduce the symbol i to denote $\sqrt{-1}$ so that $i^2=-1$. This i is not real number. It is called the fundamental imaginary unit.

Any expression of the form $(a+ib)$, where a and b are both real, is called a complex number.

The real part of the complex number $(a+ib)$ is a and is written as Re $(a+ib)$, while its imaginary part is b and is written as Img $(a+ib)$.

If $a=0$, then the complex number $(a+ib)$ becomes ib, ~~or~~ which is purely imaginary. If $b=0$, then the complex number $(a+ib)$ becomes purely real. If both $a=0$ and $b=0$, then the complex number becomes zero.

Conjugate complex number:-

If $z = a+ib$ be a complex number then its conjugate is denoted by $\bar{z}$.

If $z = a+ib$, then $\bar{z} = a-ib$.

Conjugate of conjugate of any complex number itself.
i.e., $\bar{\bar{z}} = z$.

as for example, let $z = a+ib$
then $\bar{z} = a-ib$
Now, $\bar{\bar{z}} = a+ib$.

Note:- A complex number $z = (a,b)$ may also be defined as an ordered pair of real numbers a and b such that (i) $(a,b) = (c,d)$ if and only if $a=c$ and $b=d$.
(ii) $(a,b)+(c,d) = (a+c, b+d)$
(iii) $(a,b).(c,d) = (ac-bd, ad+bc)$.
where (c,d) is another complex number

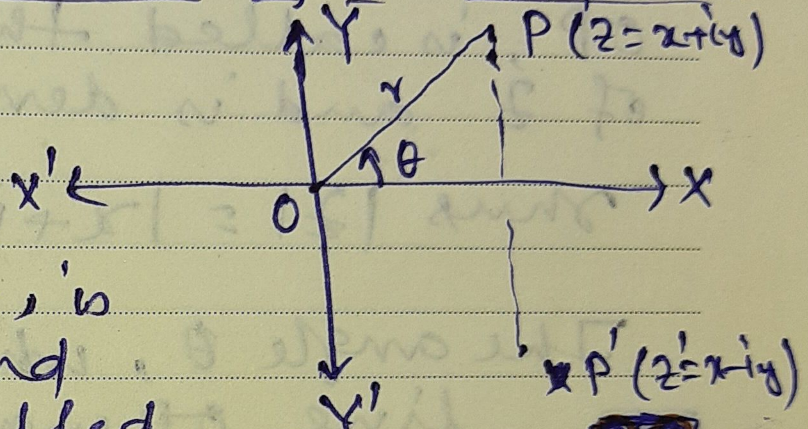
Properties of complex number:-

(a) If $a+ib=0$, a and b being real; then $a=0$ and $b=0$.
(b) If $a+ib = c+id$, then $a=c$ and $b=d$.

(c) The algebric sum, difference, product or ratio of two complex number is a complex number.

(d) The sum and the product of two conjugate complex numbers are both real.

Geometrical representions of a complex number:-



Let $z = x+iy$ be a complex number. Take, XOX' as x-axis, is called real axis and YOY' as y-axis, is called the imaginary axis.

The point $P(x,y)$ represents the complex number $z = x+iy$ and the point $P'(x,-y)$ represents the complex number $\bar{z} = x-iy$.

Polar represents of a complex number:

Let (r,θ) be the polar co-ordinates of P whose cartesian co-ordinates are (x,y) in the above figure.

Then $x = r\cos\theta$, $y = r\sin\theta$ and hence $r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

Therefore, $z = x + iy = r\cos\theta + i r\sin\theta$
$= r(\cos\theta + i\sin\theta)$

This is called the polar form of the complex number.

<u>Modulus</u> <u>and</u> ~~ad~~ <u>amplitude</u> (or <u>argument</u>):

Here r, the length of the line segment OP, is called the modulus or magnitude of z and is denoted by $|z|$ or ~~amod~~ z.

Thus $|z| = |x+iy| = r = \sqrt{x^2+y^2}$.

The angle θ, which ~~is~~ the straight line OP makes with the x-axis,

[redacted] ~~Wednesday~~ is called the amplitude or argument of z and is denoted by amp z. It has many values differing by multiple of 2π.

For the principle value of θ, we have $-\pi < \theta \leq \pi$.

The principle value of the amplitude of a complex number represented by the point P is positive acute, positive obtuse, negative obtuse or negative acute according as P lies in the first, second, third or ~~forth~~ fourth quadrant.

For example, the principle value of the amplitude of the complex number $(-1+i)$ is $\frac{3\pi}{4}$.