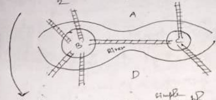
Component:

We see that a disconnected graph consist of two or more connected subgraph. Each connected subgraph are called the components.

Theorem:

A simple graph with n vertices & k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.



Proof: Let G be a simple graph with n vertices and k components. Also let the k components of G be $G_1, G_2, \ldots G_k$ with the number of vertices be $n_1, n_2, \ldots n_k$ resp.

Then $n_1+n_2+\cdots+n_k = n$ ——— (1)

Now the maximum number of edges in G_i is $\frac{n_i(n_i-1)}{2}$ ∴ $i=1,2,\ldots n$.

Therefore the maximum number of edges in G is $= \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}$

$$= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - \frac{1}{2}\sum_{i=1}^{k} n_i$$

$$= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - \frac{n}{2} \quad \text{using (1)}$$

———— (2)

Now, $\sum_{i=1}^{k}(n_i-1) = n-k$

$$\therefore \left\{\sum_{i=1}^{k}(n_i-1)\right\}^2 = (n-k)^2.$$

or $\sum_{i=1}^{k}(n_i-1)^2 + 2\sum_{\substack{i,j=1 \\ i<j}}^{k}(n_i-1)(n_j-1) = (n-k)^2$

or $\sum_{i=1}^{k}(n_i-1)^2 \leq (n-k)^2.$

Since each $n_i \geq 1$. So. $\sum_{\substack{i,j=1 \\ i<j}}^{k}(n_i-1)(n_j-1) \geq 0$

or $\sum_{i=1}^{k} n_i^2 - 2n + k \leq n^2 - 2nk + k^2$

or $\sum_{i=1}^{k} n_i^2 \leq 2n - k + n^2 - 2nk + k^2$

or $\sum_{i=1}^{k} n_i^2 \leq (2n-k) + (n-k)^2.$ ———— (3)

Using (3) in (2) we get the maximum number number of edges in G is $= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - \frac{n}{2} \leq \frac{1}{2}\left[(n-k)^2 + 2n - k\right] - \frac{n}{2}$

$$= \frac{1}{2}\left[(n-k)^2 + (n-k)\right]$$

$$= \frac{1}{2}(n-k)(n-k+1)$$

Corollary:

In a simple graph G with n vertices has more than $\frac{(n-1)(n-2)}{2}$ edges, then G is connected.

$\Rightarrow$ Let G be ~~disconnected~~ simple graph with

+ Previous theorem

n vertices & k component. Let k comp of G be $G_1, G_2, \ldots G_k$ and $n_1, n_2, \ldots n_k$ be the corresponding vertices. then we have, -

$n_1+n_2+\cdots+n_k = n$ —— ①

Now the maximum number of edges in G_i is $= \frac{n_i(n_i-1)}{2}$ $i=1,2,\ldots k$

$\therefore$ The maximum number of edges in G is $= \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2} = \frac{1}{2}\sum_{i=1}^{k}(n_i^2-n_i)$

$= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - \frac{n}{2}$

Now, $\sum_{i=1}^{k}(n_i-1) = n-k$

$\sum_{i=1}^{k}(n_i-1)^2 + 2\sum_{i=1}^{k}\sum_{j=1}^{k}(n_i-1)(n_j-1) = (n-k)^2$

$\therefore \sum_{i=1}^{k}(n_i-1)^2 \leq (n-k)^2$

$\sum_{i=1}^{k} n_i^2 - 2n+k \leq (n-k)^2$

$\sum_{i=1}^{k} n_i^2 \leq$ ~~$\frac{(n-k)(n-k+1)}{2}$~~ $(2n-k)+(n-k)^2$

Since G be a simple graph with n vertices and more than $\frac{(n-1)(n-2)}{2}$ edges. Also G has k components, then the maximum number of edges in G is - $\frac{(n-k)(n-k+1)}{2}$. But G has more than $\frac{(n-1)(n-2)}{2}$ edges, then we have

$$\frac{1}{2}(n-k)(n-k+1) > \frac{1}{2}(n-1)(n-2)$$

, which is possible only when $k=1$

$\therefore$ G is connected.

- **Complete Graph:**

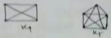
A complete graph G is a simple graph in which each pair of vertices are adjacent.

A complete graph of n vertices is denoted by K_n.

As for example:

K_2

K_3



Cycle graph:

A connected 2-regular graph is called a cycle graph. A cycle graph of n vertices is denoted by C_n

As for example:

Path graph:

A graph obtain from cycle graph by deleting an edge is called a path graph. A path graph of n vertices is denoted by P_n.

As for example:

Bipartite:

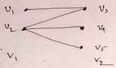
A Graph $G=(V,E)$ is said to be a bipartite graph if its vertex set V can be decomposed in to two disjoint subset V_1 & V_2 s.t every edge in G is adjacent a vertex in V_1 with a vertex in V_2.

As for example:

Let $V = \{v_1, v_2, v_3, v_4, v_5\}$

$V_1 = \{v_1, v_2\}$

$V_2 = \{v_3, v_4, v_5\}$



Complete Bipartite graph:

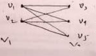
Let $G = (V, E)$ be a Bipartite graph and let V_1 & V_2 be the partition of the vertex set V of G. The Bipartite graph G is said to be complete bipartite graph if each vertex in V_1 is joined to each vertex in V_2 by exactly one edge.

As for example:

Let $V = \{v_1, v_2, v_3, v_4, v_5\}$

$V_1 = \{v_1, v_2\}$

$V_2 = \{v_3, v_4, v_5\}$



$\leftarrow K_{2,3}$