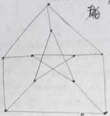


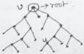
Peterson graph

- Which complete bipartite graph $K_{m,n}$ by. Euler graph?

⇒ From the defn of $K_{m,n}$ graph, that it is an Euler graph iff both m & n are even.

- <u>Tree</u>:

A connected graph without any circuit is called a tree.



Parent & children:

→ Let $u \in V(T)$ & $u \neq v$, where v is the root. Let $P(u)$ be the unique path from u to v. then the unique neighbour of u on $P(u)$ is called the parent of u. All the other neighbours of u are called the children of u.

m-ary tree:

A rooted tree is said to be m-ary tree if every non pendant vertex has atmost m children.

Stricly m-ary tree:

A rooted tree is said to be stricly m-ary tree if every non pendant vertex has exactly m children.

Binary tree:

A rooted tree is said to be ~~strict~~ binary tree if every non pendant vertex has atmost 2 children.

Strickly binary tree:

A rooted tree is said to be a Strickly Binary tree if every non pendent vertex has exactly 2 children.

is 7

Find the number of pendent vertex of a Strickly Binary tree with n-vertices.

Let G be a Strikgently binary tree with n vertices. Let $v_1, v_2, \dots v_n$ be the n vertices of G.

Since G is a tree with n vertices then n-1 edges.

Let the number of pendent vertex of G be K.

We know that by hand shaking lemma $\sum_{i=1}^{n} d(v_i) = 2 \times$ the number of edges of G

$K + 2 + 3(n-K-1) = 2 \times (n-1)$

or $K = \frac{n+1}{2}$

Theorem:

Statement:

A graph G is a tree iff there is one and only ~~go~~ path Betn any two vertices of G.

Proof: Let us assume that the graph G be a tree. Then by the defn of a tree G is a connected graph and without any circuit.

Therefore, there must exists at least one path Betn any two vertices in G.

Suppose that there are two distinct paths Betn the vertices a & b of G then the union of this two path will creat a circuit, which is contradiction. ~~Since~~

Since G has no circuit.

∴ There is one and only one path Betn any two vertices of G.

Conversely suppose that there is one and only one path Betn any two vertices of G.

We shall show that G is a tree.

Since there is a path betn any two vertices of G, So G is connected.

Let G contain a circuit. A circuit in a graph with two or more vertices implies that there exists a pair of vertices u & v s.t there are two distinct paths betn u & v, which is contradiction.

Since in G has one and only one path betn any two vertices.

So, G has no circuit. Therefore the graph G is connected with out any circuit, so G is a tree.

Theorem:

A tree with n vertices has $(n-1)$ edges.

⇒ We shall proof the theorem by induction on the number of vertices.

The theorem will be true for $n=1$

i.e $1-1=0$.

Also, the theorem is true for $n=2$

i.e $2-1 = 1$