So, the total number of edges in $G - \{e\}$ is $n_1 - 1 + n_2 - 1 = n - 2$ ($\because n_1 + n_2 = n$)

Therefore the number of edges in G is $n-1$.

Hence in a tree with n vertices has $n-1$ edges.

- **Theorem:**

Every connected graph with n vertices & $n-1$ edges is a tree.

⇒ Let G be a connected graph with n vertices and $n-1$ edges.

The theorem will be proved if we show that G has no circuit.

Suppose that G contain a circuit. Since deleting an edge from a circuit does not disconnected graph, we may removed edge but no vertices from that circuit in G, the resulting graph G^* is circuit free.

Now G^* is a connected graph with out any circuit.

Then G^* is a tree with n vertices

So G^* has $n-1$ edges.

Then, the graph G has more paths than $(n-1)$ edges, which is contradiction, since G has exactly $(n-1)$ edges.

Therefore G has no circuit.
Hence the given graph G is a tree.

Thm:
A graph G with n vertices $n-1$ edges and without any circuit is a tree.

Proof:
Let G be a graph with n vertices $n-1$ edges and no circuit.
The theorem will be proved if we show that G is connected.
Suppose that G is disconnected then G will consist of two or more circuitless components.
Let us assume that G consist exactly two components say G_1 & G_2.
We add an edge e betn a vertex v_1 in G_1 & v_2 in G_2.
Since G_1 & G_2 are in different components of G, so there is no

path both v_1 & v_2 in G

Thus addition of edge e will not creat a circuit. Thus $G \cup \{e\}$ is a circuitless, connected so $G \cup \{e\}$ is a tree.

Also $G \cup \{e\}$ has n vertices and n edges, which is contradiction. Since in a tree with n vertices then n-1 edges.

$\therefore$ the graph G is connected

Hence G is a tree.

Planar Graph:

A graph G is said to be planner if the graph can be drawn in a plane s.t no edges cross each other. as for example K_4 is a planner graph.

$n-e+f=2$

Let us assume that the theorem is true for all the vertices up to $n-1$

Let us assume that a tree G with n vertices

Let e be any edge in G with end vertices v_i & v_j.

Then the edge e is the only one path bet v_i & v_j. Now deleting the edge e from G, then the resulting graph must be disconnected. i.e $G-\{e\}$ is a disconnected graph. Also $G-\{e\}$ will contain exactly two components, for otherwise the graph G will not be connected.

Let this two components of $G-\{e\}$ be G_1 & G_2. Each of this component is a tree. Because there were no circuit in G.

Let n_1 & n_2 be the number of vertices in G_1 & G_2 resp.

Since $n_1 < n$ & $n_2 < n$, then by our assumption the number of edges in G_1 is n_1-1 & the number of edges in G_2 is n_2-1