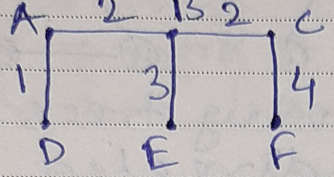
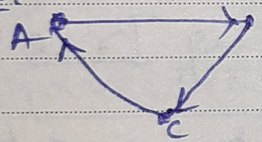
Weighted graph:- A weighted graph is a graph in which a non-negative real number is assigned to each edge. The number assigned to ~~...~~ each edge e is called weight of e and it is denoted by w(e).

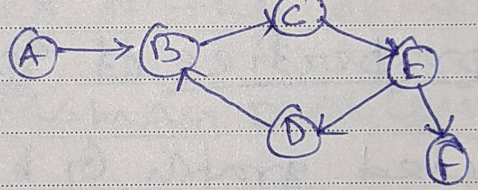


W(AD) = 1, W(AB) = 2 = W(BC)
W(BE) = 3, W(CF) = 4.

Directed graph:- A directed graph is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them.

Ex:-



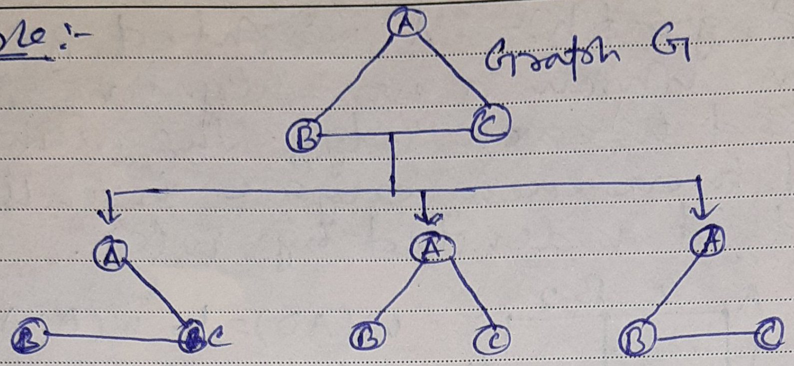


Spanning tree:- A spanning tree is a subset of graph G, which has all the vertices covered with minimum possible number of edges.

Hence a ~~s~~ spanning tree does not have cycles and it can not be disconnected.

By this definition, we can draw a conclusion that every connected ~~graph~~ and undirected graph G has atleast one spanning ~~tr~~ tree. A disconnected graph does not have any spanning tree, ~~a~~ as it ~~...~~ can not be spanned to all its vertices.

Example:-



We found three spanning trees of one complete graph. A complete undirected graph can have ~~a~~ maximum n^{n-2} number of spanning trees, where n is the number of nodes. In the ~~a~~ above example, n is 3, hence $3^{3-2} = 3$ spanning trees are possible.

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General properties of Spanning Tree:-

(i) A connected graph G can to have ~~been~~ more than one spanning tree.
(ii) All possible spanning trees of graph G, have the same number of edges and vertices.
(iii) The spanning ~~tree~~ tree does not have any cycle (loops).
(iv) Removing one edge from the spanning tree will make the graph disconnected, i.e., the spanning tree is minimally connected.
(v) Adding one edge to the spanning tree will creat a circuit or loop, i.e., the spanning tree is maximally acyclic.

Mathematical properties of Spanning Tree:-

(i) Spanning tree has (n-1) edges, where n is the number of nodes (vertices).

(ii) From a complete graph, by removing maximum $(e-n+1)$ edges, we can construct a spanning tree.

(iii) A complete graph can have maximum n^{n-2} number of spanning trees.

Thus a spanning trees are a subset of connected graph G and disconnected graphs do not have spanning tree.

Minimum spanning Tree ~~ss~~ (MST):-

In a weighted graph, a minimum ~~span~~ spanning tree is a spanning tree that has minimum weight than all other spanning trees of the same graph.

Travelling salesman problem:-

Travelling ~~sallss~~ salesman problem, an optimization problem in graph theory in which the nodes (cities) of a graph are connected by directed edges (routes), where the weight of an edge indicates the distance between two cities.

The problem is to find a path that visits each city once, returnes to the starting city, and minimizes the distance traveled.

Matrix Representation of Graph

Adjacency matrix:-

Defn:- Let $G = (V, E)$ be a undirected graph with no multiple edges where $V = \{1, 2, \text{--} n\}$. The adjacency matrix of G is the $n \times n$ matrix $A = (a_{ij})$ where (i) $a_{ij} = 1$ if there is an edge betn vertex i and vertex j.
and (ii) $a_{ij} = 0$ otherwise.

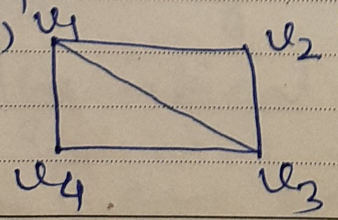
Observation:- (i) A is symmetric if $a_{ij} = a_{ji}$ $\forall i$ and j.
(ii) The ~~end~~ entries along the principle diagonal of A all zeros iff the graph has no self loop
(iii) If the graph is simple (no self loop,

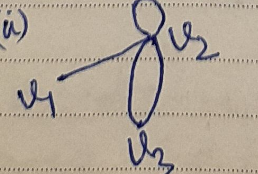
no parallel edge) the degree of the vertex equal to the number of one's in the corresponding row and column of A.

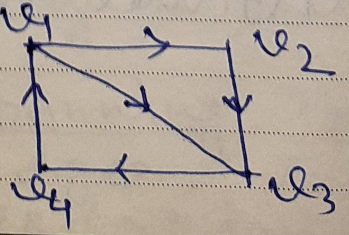
Representation of Directed graph:-

The adjacency matrix of a directed graph D with n vertices is the matrix $A = (a_{ij})_{n \times n}$ whose element are given by
(i) $a_{ij} = 1$, if are (v_i, v_j) ~~as~~ is in D.
$= 0$, otherwise.

Ex:(1) Find adjacency matrix represented the graph.

(i)


(ii)


(iii)


Sol:- (i) We order the vertices as v_1, v_2, v_3, v_4. Since there are four vertices, the adjacency matrix representing the graph will be a square matrix of order four.

The required adjacency matrix is

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

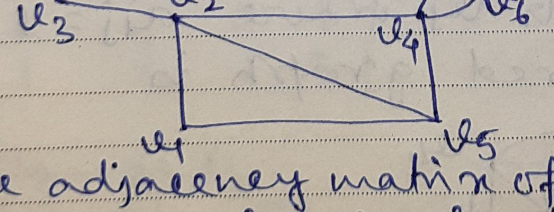
(ii) The required adjacency matrix of order 3 is

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} \end{array}$$

(iii) The adjacency matrix of order four is

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

2. Find the adjacency matrix of the graph



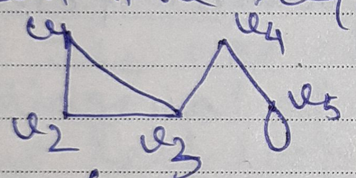
Sol:- The adjacency matrix of order six is

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

3. Draw the undirected graph represented by the adjacency matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Sol:- Since the given matrix is ~~of~~ a square matrix of order five, The graph G has five vertices v_1, v_2, v_3, v_4, v_5.
Let us draw and edge from v_i to v_j where $a_{ij}=1$.
Then the required graph is



G

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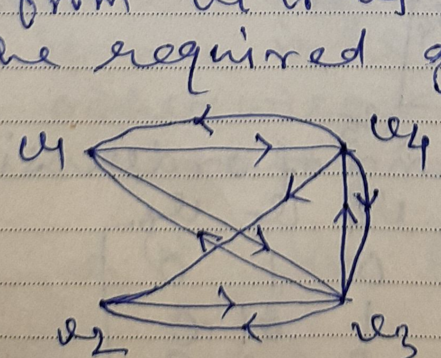
4. Draw the directed graph G corresponding to the adjacency matrix.

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Sol:- Since the given matrix is a square matrix of order four The graph G has four vertices v_1, v_2, v_3, v_4. Let us draw an edge from v_i to v_j where $a_{ij} = 1$.
Then the required graph is



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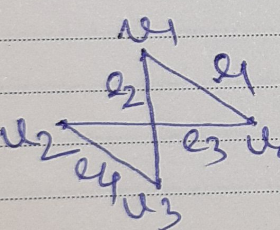
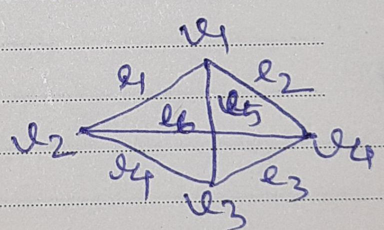
Incidence matrix:-

Def :- The incidence matrix of an undirected graph is a $n \times m$ matrix B, where n and m are the numbers of vertices and edges respectively, such that $B_{ij} = 1$ if the vertex v_i and edge e_j are incident and 0 otherwise.

The incidence matrix of a directed graph is a $n \times m$ matrix B where n and m are the number of vertices and edges respectively, such that $B_{ij} = -1$ if the edge e_j leaves vertex v_i, 1 if it enters vertex v_i and 0 otherwise.

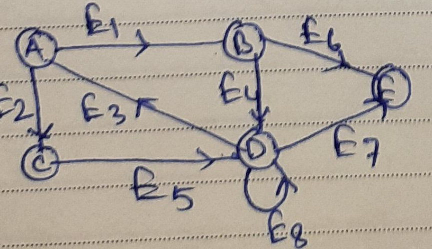
Ex:-





$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array}$$



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{cccccccc} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix} \end{array}$$