Semester - VI

Course Type - DSE-4

Course Title: DSE-4: Mathematics Modelling.

Topic: Home Assignment on L.T.

Date: 23.04.2020

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Laplace Transformation
(Assignment)-1

1. Prove that $L^{-1}\left\{\frac{1}{(p+2)^2(p-2)}\right\} = \frac{1}{16}\left(e^{2t} - 4te^{-2t} - e^{-2t}\right)$

2. find value of $\int_0^\infty \frac{e^{-3t} - e^{6t}}{t}\,dt$.

3. Find $L^{-1}\left\{\frac{1+2s}{(s+2)^2(s-1)^2}\right\}$

4. prove that $L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}$.

5. Use convolution theorem to find
$L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$

6. Solve the boundary value problem
$y'' + 2y' + y = 0$, given $y(0) = 0$ and $y(1) = 2$, when $y'' = \frac{d^2y}{dt^2}$ and $y' = \frac{dy}{dt}$.

7. Solve $(D^2 + 2D + 5)y = e^{-t}\sin t$, $y(0) = 0$, $y'(0) = 1$.

8. Solve $\frac{\partial y}{\partial x} = y + 2\frac{\partial y}{\partial t}$, $y(x,0) = 6e^{-3x}$, which is bounded for $x > 0$, $t > 0$.

9. Solve $3\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ where $y(0,t) = 0$, $y(5,t) = 0$ and $y(x,0) = -5\sin 6\pi x$.

10. prove that $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\, dt = \log\left(\frac{2}{3}\right)$.

11. Find the Laplace transformation of $\frac{\sin at}{t}$. Does the Laplace transform of $\frac{\cos at}{t}$ exist?

12. Evaluate $L\left\{\frac{e^{-at}\, t^{n-1}}{(n-1)!}\right\}$

13. If $f(s)$ be the Laplace transform of $F(t)$ then $L\left\{\int_0^t F(T)\, dT\right\} = \frac{f(s)}{s} = \frac{1}{s} L\{F(t)\}$.

14. Solve $Dx + Dy = t$, $D^2x - y = e^{-t}$ if $x(0) = 3$, $x'(0) = -2$, $y(0) = 0$.

15. solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial t^2}$, $x > 0$, $t > 0$ given that $y(0,t) = 1$ and $y(x,0) = 0$