Semester-IV

Course Type:- Core-8

Course Title: C8T: Sequence of function

Topic: Home Assignment on sequence of function.

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Sequence of Function (Assignment)

Theorem:-

1. A necessary and sufficient condition for uniform convergence of a sequence $\{f_n\}$ on D is that for a pre-assigned positive ϵ $\exists$ a natural number K such that for all $x \in D$, $|f_m(x) - f_n(x)| < \epsilon$ $\forall\, m, n \geq K$.

2. **Theorem:** Let D be a subset of $\mathbb{R}$ and a sequence of functions $\{f_n\}$ be uniformly convergent on D to a function f. Let $x_0 \in D'$ and $\lim_{x \to x_0} f_n(x) = a_n$. Then
   (i) the sequence $\{a_n\}$ is convergent and
   (ii) $\lim_{x \to x_0} f(x)$ exists and equals $\lim_{n \to \infty} a_n$.

3. Let $D \subset \mathbb{R}$ and for each $n \in \mathbb{N}$, $f_n : D \to \mathbb{R}$ is a continuous on D. If the sequence $\{f_n\}$ be uniformly convergent on D to a function f, then f is continuous on D.

4. prove that the sequence $\{f_n\}$ defined by $f_n(x) = \frac{nx}{1+nx}$, $x \geq 0$ is not uniformly convergent on $[0, \infty)$ but the convergence is uniform on $[a, \infty)$ if $a > 0$

(5) Let $f_n(x) = nxe^{-nx^2}$, $x \in [0,1]$. Show that the sequence $\{f_n\}$ is not uniformly convergent on $[0,1]$.

(6) Examine uniform convergence of the sequence $\{f\}$ on $[0,1]$ defined by (i) $f_n(x) = \dfrac{nx}{1+n^3x^2}$, $x \in [0,1]$;

(ii) $f_n(x) = n^2x^2e^{-nx}$, $x \in [0,\infty)$

(iii) $f_n(x) = nx(1-x)^n$, $x \in [0,1]$.

(7) Let $f_n(x) = nx^2$, $0 \leq x \leq \frac{1}{n}$

$= x$, $\frac{1}{n} < x \leq 1$.

Show that $f_n(x)$ is converges to a function f on $[0,1]$.

(8) Examine uniform convergence of $f_n(x)$ defined by $f_n(x) = n^2x(1-x^2)^n$, $0 \leq x \leq 1$.

(9) Let $I = [a,b]$ be a closed and bounded interval and for each $n \in N$, $f_n : I \to \mathbb{R}$ be R-integrable on I. If the sequence $\{f_n\}$ converges uniformly to a function f on I then f is R integrable on I and moreover, the sequence $\{\int_a^b f_n\}$ converges to $\int_a^b f$.