E - LEARNING MATERIALS

SEM - 6 , CC - 13 .

Unit - 1, 2.

Topic - Metric spaces. (Home Assignment) - 1

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Questions.

1. Let X be a set of all continuous real valued functions defined on $[0,1]$ and let $d(x,y) = \int_0^1 |x(t) - y(t)|\, dt$, $x, y \in X$. Show that (X, d) is not a complete metric space. (1)

2. Let (X, d) be a complete metric space and Y be a subspace of X. Prove that Y is complete if and only if it is closed in (X, d) [It is a theorem, prove it].

3. Prove that the open interval $(0,1)$ with usual metric is not compact.

4. Let X be an infinite set with discrete metric. Show that (X, d) is not compact.

5. Show that a continuous function may not send a Cauchy sequence to a ~~Ca~~ Cauchy sequence.

6. Show that every closed sub set of a compact metric space is compact. [Theorem].

7. Use 6, show that $\mathbb{N}$ is not compact in $\mathbb{R}$.

8. Show that every compact sub set of F of a metric space (X, d) is closed.

9. Every compact sub set of a metric space is closed and bounded. Give an example to show that the converse of the statement is not true.

10. State and prove Heine-Borel theorem.

11. The converse of the Heine-Borel

theorem is true. Prove it. (3)

11. Show by an example that the Heine-Borel theorem does not hold good in a general metric space.

12. Show that continuous image of a compact set is compact.

13. State FIP and BWP. Define sequentially compact metric space.

14. Show that a metric space (X, d) is compact if and only if every family of closed sets in (X, d) with FIP has a non-empty intersection.

15. Show that a metric space (X, d) is compact if every

sequence $\{x_n\}$ in X has a convergent sub sequence.

16. Show that every compact metric space (X,d) is sequentially compact.

17. Define totally bounded set. Show that an infinite discrete space is bounded but not totally bounded.

18. Show that a metric space (X,d) is sequentially compact if and only if it is complete and totally bounded.

<u>Note</u>: Prove the theorem 30, 31, 32, 33 from Page no. 796, 797.