Sun	Mon	Tue	Wed	Thu	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

② Solution of the first order Partial Differential Equation $Pp+Qq=R$ (Lagrange's Method)

Here $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$ and P, Q, R are functions of x, y and z. Also here x, y are two independent variables and z is the dependent variable.

The equⁿ $Pp+Qq=R$ — ① is the corresponding partial differential equⁿ of a function $\phi(u,v)=0$ — ② where u and v are two functions of x, y and z.

Let us consider two independent integrals given by $u(x,y,z)=a$

and $v(x,y,z)=b$

where a and b are two arbitrary constants.

Therefore $du=0$ and $dv=0$. Saturday 14

These gives,

$$\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy+\frac{\partial u}{\partial z}dz=0$$

and $$\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy+\frac{\partial v}{\partial z}dz=0$$

from these, we get

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R} \quad \text{—— ③}$$

where P, Q and R are the function of x, y, z.

Thus $u(x,y,z)=a$ and $v(x,y,z)=b$ are the solⁿs of the differential equation ③.

This equⁿ (3) is known as the subsidiary equation or auxiliary equation.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Thus to solve an equ^n $Pp+Qq=R$, we first solve $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ and get its two independent sol^{n}s $u=$ constant and $v=$ constant. Then the general sol^n of $Pp+Qq=R$ will be $v=\phi_1(u)$ or $\phi(u,v)=0$ where ϕ_1 or ϕ is any arbitrary functions.

<u>Note</u>:- If we get any integral which satisfies the equ^n (1) but it cannot be expressed as a function of u and v, then this type of solution is known as <u>Special solution</u>.

<u>Solved Example</u>:-

Q1. Solve $xp+yq=z$.

<u>Sol^n</u>:- The above equ^n is of the form
$$Pp+Qq=R$$

The Lagrange's subsidiary equ^{n}s are
$$\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z} \quad \text{——— (i)}$$

from first two parts of (i) we see
$$\frac{dx}{x}=\frac{dy}{y} \text{ i.e., } \frac{dx}{x}-\frac{dy}{y}=0$$

Integrating, $\log x-\log y=\log c_1$ ——— (ii)

or, $\frac{x}{y}=c_1$, c_1 is an arbitrary const.

Again taking first and third parts, we see

$$\frac{dx}{x} = \frac{dz}{z}$$

or, $\frac{dx}{x} - \frac{dz}{z} = 0$

Integrating, $\log x - \log z = \log c_2$, c_2 being an arbitrary const.

or, $\frac{x}{z} = c_2$. —(iii)

The general solⁿ of the equⁿ is

$$\phi\left(\frac{x}{y}, \frac{x}{z}\right) = 0.$$

ϕ being an arbitrary function

Q2. Solve $xp - yq = xy$.

Sol:- The above equⁿ is of the form

$$Pp + Qq = R.$$

Therefore the Lagrange's subsidiary equⁿs are $\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{xy}$ —(i)

Taking first two parts of (i) we see

$$\frac{dx}{x} = \frac{dy}{-y}$$

or, $\frac{dx}{x} + \frac{dy}{y} = 0.$

Int., $\log x + \log y = \log c_1$, c_1 be an arbitrary const.

or, $xy = c_1$ —(ii)

Again taking the first and third parts of (i) we see $\frac{dx}{x} = \frac{dz}{xy}$

or, $\frac{dx}{x} = \frac{dz}{c_1}$ [using (ii)]

or, $c_1 \frac{dx}{x} = dz$

Int,

$c_1 \log x = z + c_2$, c_2 being an arbitrary const.

or, $xy \log x - z = c_2$ [using (ii)] —— (iii)

Using (ii) and (iii), the general solⁿ of the equⁿ is given by

$xy \log x - z = \phi(xy)$,

ϕ being an arbitrary function.

Q3. Solve $x^2 p + y^2 q = (x+y) z$

Sol:- The above equⁿ is of the form
$Pp + Qq = R$.

So, the Lagrange's subsidiary equⁿs are

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{(x+y)z} \quad \text{—— (i)}$$

Taking first two parts of (i) we get

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

or $\frac{dx}{x^2} - \frac{dy}{y^2} = 0$

Int. $\frac{1}{x} - \frac{1}{y} = c_1$, c_1 being an arbitrary const. —— (ii)

Again each part of (i) is equal to

$$\frac{-\frac{1}{x}dx - \frac{1}{y}dy + \frac{1}{z}dz}{-x-y+x+y} = \frac{-\frac{1}{x}dx - \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

i.e., $-\frac{1}{x}dx - \frac{1}{y}dy + \frac{1}{z}dz = 0$

Int., $\log \frac{z}{xy} = \log c_2$, c_2 be an arbitrary const.

or $\frac{z}{xy} = c_2$ —(iii)

Therefore the general sol^n of the equ^n using (ii) and (iii) is given by

$$\phi\left(\frac{1}{x} - \frac{1}{y}, \frac{z}{xy}\right) = 0.$$

ϕ being an arbitrary fun^n ~~Sunday~~ ~~22~~

Q4. Solve $p + 3q = 5z + \tan(y - 3x)$.

Sol^n:- The given equ^n is of the form

$$Pp + Qq = R.$$

So, the Lagrange's subsidiary equ^n will be

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y-3x)} \quad \text{—(i)}$$

Taking first two from (i) we get

$$dy - 3dx = 0$$

Int. we get $y - 3x = c_1$, c_1 being an arbitrary const. —(ii)

Again from first and third parts we get

$$dx = \frac{dz}{5z + \tan(y-3x)}$$

or, $dx = \frac{dz}{5z + \tan c_1}$ [using (ii)]

Int., we get,

$x = \frac{1}{5}\log(5z + \tan c_1) + \frac{1}{5}c_2$, c_2 be an arbitrary const.

or, $5x - \log[5z + \tan(y-3x)] = c_2$ ... (iii)

Using (ii) and (iii), the required general solⁿ will be given by

$$5x - \log[5z + \tan(y-3x)] = \phi(y-3x),$$

where ϕ is an arbitrary ~~constant~~ function.

24 ~~Tuesday~~

Solve the following Problem.

1. Solve $p + q = x + y + z$.

2. Solve $zp + x = 0$.

3. Solve $2xp + 2yq = xy$.

4. Solve $(y + zx)p - (x + yz)q + y^2 - x^2 = 0$.