E - learning Materials.

Sem - 4. CC - 08

Unit - 1

Topic - Riemann Integration.
(Home Assignment) - 1

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Questions.

1. Let $f : [0,1] \longrightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x, & x \in [0,1] \cap \mathbb{Q} \\ 0, & x \in [0,1] \setminus \mathbb{Q}. \end{cases}$$

Show that $f \in \mathcal{R}[0,1]$.

2. Let $f : [0,1] \longrightarrow \mathbb{R}$ be defined by

$$f(x) = 2x, \quad x \in [0,1] \cap \mathbb{Q}$$
$$= 1-x, \quad x \in [0,1] \setminus \mathbb{Q}.$$

Show that $f \notin \mathcal{R}[0,1]$.

Evaluate (i) $\lim\limits_{n \to \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+3n} \right]$

(ii) $\lim\limits_{n \to \infty} \left[\frac{n}{n^2+1} + \frac{n}{n^2+2^2} + \cdots + \frac{n}{n^2+4n^2} \right]$

Using 1st Mean Value Theorem prove that

$$\frac{16\sqrt{2}}{41\pi} \leqslant \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{\sin \pi x}{2+x^2}\, dx \leqslant \frac{16\sqrt{2}}{33\pi}$$

5. Using 1st Mean value theorem prove that

$$\frac{\pi}{6} \leq \int_0^{\frac{1}{2}} \frac{1}{\sqrt{(1-x^2)(1-k^2x^2)}}\,dx \leq \frac{\pi}{6} \cdot \frac{1}{\sqrt{1-\frac{k^2}{4}}} ; (k^2<1)$$

6. State 2nd mean value theorem (Bonnet's form) and use it to show that

$$\left|\int_a^b \frac{\sin x}{x}\,dx\right| \leq \frac{2}{a}, \quad 0<a<b<\infty$$

7. Using 2nd mean value theorem (Weierstrass form) and use it to show that

$$\left|\int_a^b \frac{\cos x}{1+x}\,dx\right| < \frac{4}{1+a}, \quad 0<a<b<\infty$$

8. Verify 2nd MVT of Integral Calculus for the integral $\int_{-\pi}^{\pi} x^2 \cos x\,dx$.

9. If f is continuous on $[a,b]$, is f is necessarily to integrable on $[a,b]$? Justify.

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