Theorem on Multiple roots

Theorem:- If α be a multiple root of the eqn $f(x)=0$ of order r, then α is a multiple root of the eqn $f'(x)=0$ of order $r-1$.

Theorem:- If α be a multiple root of the eqn $f(x)=0$ of order r then
$f(\alpha)=f'(\alpha)=f''(\alpha)=\cdots=f^{r-1}(\alpha)=0$
and $f^{r}(\alpha)\neq 0$.

Ex:- The eqn $ax^3+3bx^2+3cx+d=0$ has two equal roots. Prove that $(bc-ad)^2=4(b^2-ac)(c^2-bd)$ and the equal root is $\frac{1}{2}\frac{(bc-ad)}{ac-b^2}$.

Soln:- Let $f(x)=ax^3+3bx^2+3cx+d$ and α be a double root of $f(x)=0$.
Then $f(\alpha)=0$, $f'(\alpha)=0$

Therefore. $a\alpha^3+3b\alpha^2+3c\alpha+d=0$ —(i).

and $3a\alpha^2+6b\alpha+3c=0$

or, $a\alpha^2+2b\alpha+c=0$ —(ii)

Multiplying (ii) by α and subtracting from (i) we get

$b\alpha^2+2c\alpha+d=0$ —(iii)

From (ii) and (iii), by cross multiplication we get

$$\frac{\alpha^2}{2(bd-c^2)}=\frac{\alpha}{bc-ad}=\frac{1}{2(ac-b^2)}$$

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or, $\alpha^2 = \dfrac{bd - c^2}{ae - b^2}$ and $\alpha = \dfrac{bc - ad}{2(ae - b^2)}$

Therefore, or, $\alpha^2 = \dfrac{(bc - ad)^2}{4(ae - b^2)^2}$

$$\frac{bd - c^2}{ae - b^2} = \frac{(bc - ad)^2}{4(ae - b^2)^2}$$

or, $(bc - ad)^2 = 4(b^2 - ae)(c^2 - bd)$ [Proved]

Also $\alpha = \dfrac{bc - ad}{2(ae - b^2)}$

which is the equal root.

30 Saturday **Ex:-** If the eqn $x^4 + px^2 + qx + r = 0$ has three equal roots then show that $8p^3 + 27q^2 = 0$ and $p^2 + 12r = 0$.

Theorem:- If an eqn with ~~root~~ real co-efficients has a complex root $\alpha + i\beta$ then it has also a conjugate complex root $\alpha - i\beta$.

Ex:- Solve the eqn $x^4 + x^2 - 2x + 6 = 0$; it is given that $1+i$ is a root.

Soln:- Let $f(x) = x^4 + x^2 - 2x + 6$.
Since $f(x) = 0$ is eqn with real co-efficients and $1+i$ is a root of the eqn, $1-i$ is also a root.

Therefore $(x-1-i)(x-1+i) = x^2-2x+2$ is a factor of $f(x)$.

Let $f(x) = (x^2-2x+2)\,q(x)$.

Again clearly,

$$x^4+x^2-2x+6 = (x^2-2x+2)(x^2+2x+3)$$

Therefore $q(x) = x^2+2x+3$

$q(x)=0$ gives $x = \dfrac{-2\pm\sqrt{4-4\cdot1\cdot3}}{2}$

or, $x = \dfrac{-2\pm2\sqrt{-2}}{2}$

$= -1\pm i\sqrt{2}$

Therefore the roots are $1\pm i$, $-1\pm i\sqrt{2}$

__Theorem:-__ If an eqn with rational co-efficients has a surd root $\alpha+\sqrt{\beta}$, where α, β are rational and β is not a perfect square, then it has also the conjugate root $\alpha-\sqrt{\beta}$.

~~Ex:- Solve the eqn~~

__Ex:-__ For a biquadratic eqn with rational co-efficients two of whose roots are $\sqrt{3}\pm2$.

__Sol:-__ Since the eqn have roots $\sqrt{3}\pm2$ i.e, $2+\sqrt{3}$ and $-2+\sqrt{3}$, then the two roots of the eqn are $2-\sqrt{3}$ and $-2-\sqrt{3}$.

Therefore, the ~~equⁿ~~ required equⁿ is

$(x-\sqrt{3}-2)(x-\sqrt{3}+2)(x+\sqrt{3}-2)(x+\sqrt{3}+2)=0$

or, $\{(x-\sqrt{3})^2-4\}\{(x+\sqrt{3})^2-4\}=0$.

or, $\{(x^2-1)-2\sqrt{3}x\}\{(x^2-1)+2\sqrt{3}x\}=0$

or, $(x^2-1)^2-12x^2=0$

or, $x^4-14x^2+1=0$

which is the reqⁿ equⁿ.

Ex:- Solve the equⁿ $x^4+2x^3-16x^2-22x+7=0$ which has a root $2+\sqrt{3}$.

4 ~~Wednesday~~

Rolle's theorem:- Let $f(x)$ be a polynomial of real coefficients. ~~But~~ Between any two consecutive real roots of the equⁿ $f(x)=0$ there is atleast one real root of the equⁿ $f'(x)=0$.

Note:- If between two consecutive real roots of the equⁿ $f(x)=0$ there is more than one real root of the equⁿ $f'(x)=0$ then the number of roots are odd.

Descarte's rule of sign:-

Statement:- The number of positive roots of an equ^n $f(x)=0$ with real co-efficients does not exceed the number of variation of singns in the sequence of the coefficients of $f(x)$ and if less. it is less by an even number.

The number of negative roots of an equ^n $f(x)=0$ with real co-efficients does not exceed the number of variation of signs in the sequence of the coefficients of $f(-x)$ and if less, it is less by an even number.

Ex:- Appy Descarte's rule of signs to find the nature of the roots of the equ^n $x^4+2x^2+3x-1=0$.

Sol^n:- Let $f(x)=x^4+2x^2+3x-1$

Then $f(-x)=x^4+2x^2-3x-1$

The signs in the sequence of coefficients of $f(x)$ are $+\ +\ +\ -$. There is only one variation of signs and therefore the number of positive roots of $f(x)=0$ is exactly 1.

The signs in the sequence of coefficients of $f(-x)$ are $+\ +\ -\ -$. There is only one variation of signs and therefore the number of

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negative roots of $f(x)=0$ is exactly 1. The eqn has no zero roots. Therefore the number of real roots is 2. The eqn being of degree 4 has 4 roots. Consequently, the number of complex root of the eqn is 2.

(iii) ~~Sol:~~ ~~x^7+x^5~~

Ex:- Apply Descarte's rule of sign to find the nature of the roots of the eqn $x^7+x^5-x^3=0$

8 ~~Sunday~~

Sol: $f(x) = x^7+x^5-x^3$.

$f(-x) = -x^7-x^5+x^3$

signs of coefficients of $f(x)$ are $+ + -$

since 1 variation so exactly 1 positive roots

signs of coefficient of $f(-x)$ are $- - +$

since 1 variation, so exactly 1 negative root

The eqn has 3 zero roots. Therefore the number of real root is 5. The eqn being of degree 7 has 7 roots. Consequently, the number of complex root of the eqn is 2.