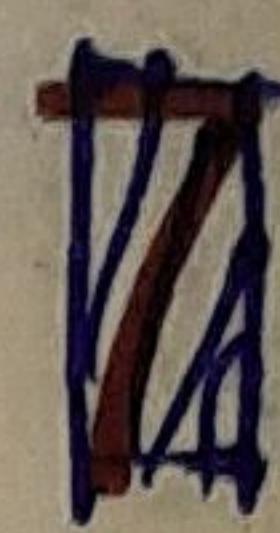


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Wednesday



VECTOR CALCULUS

Dot product or Scalar product:-

Let A and B be two vectors. Then dot product or scalar product between them is

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta, \quad 0^\circ \leq \theta \leq 180^\circ$$

where θ is the angle between the two vectors.

(R) Suppose A , B and C are vectors and m is scalar. Then the following laws hold.

(i) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$, commutative law for dot product

(ii) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$, distributive law.

(iii) $m(\vec{A} \cdot \vec{B}) = (m\vec{A}) \cdot \vec{B} = \vec{A} \cdot (m\vec{B}) = (\vec{A} \cdot \vec{B})m$

(iv) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

~~Thursday~~ [where i, j, k be the unit vector corresponding to the rectangular system.]

(v) If $\vec{A} \cdot \vec{B} = 0$ and A and B are not null vector, then A and B are perpendicular.

Let $\vec{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k}$, $\vec{B} = B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}$

$$\therefore \vec{A} \cdot \vec{B} = A_1 B_1 + A_2 B_2 + A_3 B_3.$$

Cross product:-

The cross products of vectors A and B is a vector $C = A \times B$ and θ be the angle of the two vector A and B .

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The direction of $c = A \times B$ is perpendicular to the plane of A and B so that A, B and c form a right handed system.

$$\vec{A} \times \vec{B} = |A||B| \sin\theta \cdot \hat{n} \quad [\hat{n} \text{ is the unit vector in the direction of } \vec{A} \times \vec{B}]$$

If $\vec{A} = \vec{B}$ or if A is parallel to B , then $\sin\theta = 0$ and hence $\vec{A} \times \vec{B} = \vec{0}$.

④ Suppose \vec{A}, \vec{B} and \vec{C} are vectors and m is a scalar, then the following laws holds.

$$(i) \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad [\text{commutative law doesn't hold}]$$

$$(ii) \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad [\text{Distributive law}]$$

$$(iii) m(\vec{A} \times \vec{B}) = (\vec{A} \times \vec{B})m = \vec{A} \times (m\vec{B}) = (\vec{A} \times \vec{B})m$$

~~Wednesday~~ (iv) $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$
 $\vec{i} \times \vec{j} = \vec{k}, \vec{j} \times \vec{k} = \vec{i}, \vec{k} \times \vec{i} = \vec{j}$
 $\vec{j} \times \vec{i} = -\vec{k}, \vec{k} \times \vec{j} = -\vec{i}, \vec{i} \times \vec{k} = -\vec{j}$.

(v) If $\vec{A} \times \vec{B} = 0$ and A and B are not null vector, then A and B parallel.

④ Given $\vec{A} = A_1 \vec{i} + A_2 \vec{j} + A_3 \vec{k}$
and $\vec{B} = B_1 \vec{i} + B_2 \vec{j} + B_3 \vec{k}$

then $\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$

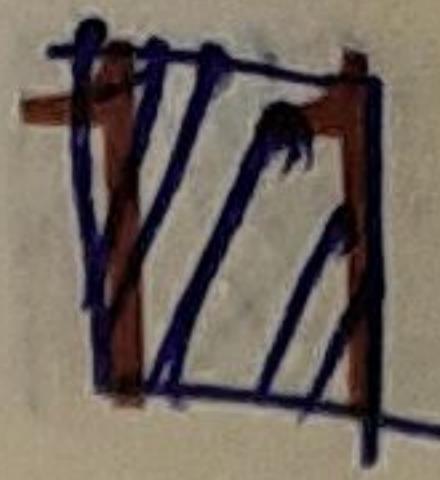
$$= \vec{i}(A_2 B_3 - A_3 B_2) + \vec{j}(A_3 B_1 - A_1 B_3) + \vec{k}(A_1 B_2 - A_2 B_1)$$

December

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November 2018

~~Monday~~

Product of three vector (Triple product)

from three vector A, B and C we may obtain

1. $\alpha A \cdot (B \times C)$, which is a scalar.

This is known as Scalar Triple Product of A, B, C.

2. $A \times (B \times C)$, which is a vector.

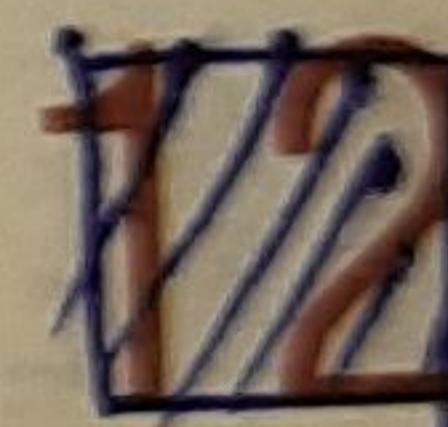
This is known as vector Triple product

Note:- $\alpha(B \times C)$ is a vector α

$A \cdot (B \cdot C)$ is a vector A multiplied by a scalar B.C.

$A \times (B \cdot C)$ has no meanings.

Scalar Triple Product :-

~~Monday~~

Let A, B, C be three vectors with component (a_1, a_2, a_3) , (b_1, b_2, b_3) and (c_1, c_2, c_3) respectively with respect to the fundamental rectangular system i, j, k .

Then we have:

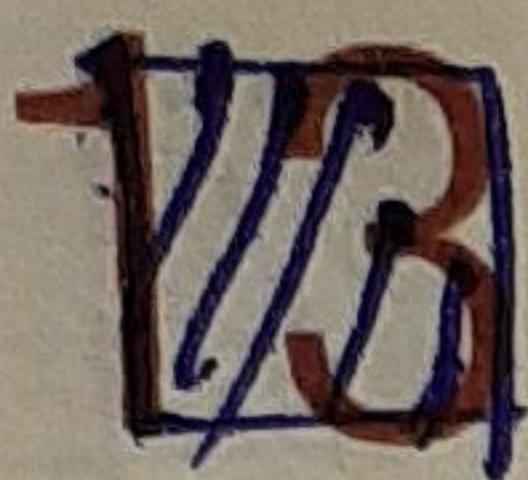
$$\vec{B} \times \vec{C} = \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= i(b_2 c_3 - b_3 c_2) + j(b_3 c_1 - b_1 c_3) + k(b_1 c_2 - b_2 c_1)$$

and hence scalar triple product

$$A \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= a_1(b_2 c_3 - b_3 c_2) + a_2(b_3 c_1 - b_1 c_3) + a_3(b_1 c_2 - b_2 c_1)$$



Tuesday

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The scalar triple product $a \cdot A \cdot (B \times C)$ is also written as $[A B C]$

Note:- If a, b, c be three vectors. then

$$\begin{aligned} 1. \quad b \cdot (c \times a) &= (c \times a) \cdot b = -b \cdot (a \times c) = -(a \times c) \cdot b \\ c \cdot (a \times b) &= (a \times b) \cdot c = -c \cdot (b \times a) = -(b \times a) \cdot c \\ a \cdot (b \times c) &= (b \times c) \cdot a = -a \cdot (c \times b) = -(c \times b) \cdot a \end{aligned}$$

In box notation,

$$\begin{aligned} [abc] &= [bca] = [cab] = -[bac] \\ &= -[acb] \text{ etc.} \end{aligned}$$

2. If three non-zero vectors a, b, c are co-planar, then their scalar triple product is zero i.e. $[abc] = 0$.

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Wednesday

3. The scalar triple product $[abc]$ is numerically equal to the volume V of a parallelopiped having a, b, c as concurrent edges.

4. The relation of the vectors of the fundamental system ($i-j-k$ system) as

$$[\hat{i} \hat{j} \hat{k}] = [\hat{j} \hat{k} \hat{i}] = [\hat{k} \hat{i} \hat{j}] = 1$$

$$\text{and } [\hat{i} \hat{k} \hat{j}] = [\hat{k} \hat{j} \hat{i}] = [\hat{j} \hat{i} \hat{k}] = -1.$$

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~~Thursday~~

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Vector triple product:-

we now consider the cross product of

or a and $b \times c$ i.e., $a \times (b \times c)$

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c.$$

Corollary:-

$$1. b \times (c \times a) = (b \cdot a)c - (b \cdot c)a = (a \cdot b)c - (b \cdot c)a$$

$$2. a \times (b \times c) = -c \times (a \times b) = -[(c \cdot b)a - (c \cdot a)b] \\ = (c \cdot a)b - (c \cdot b)a.$$

3. $a \times (b \times c)$, $b \times (c \times a)$ and $c \times (a \times b)$ lies on the same plane.

Example:-

Q. Let $a = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $b = -5\hat{i} + 7\hat{j} - 3\hat{k}$
 $c = 7\hat{i} - 5\hat{j} - 3\hat{k}$. Find $a \times (b \times c)$ in terms of $\hat{i}, \hat{j}, \hat{k}$ and then verify
 $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$.

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$$\text{Sol: } b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix} = -36\hat{i} - 86\hat{j} - 24\hat{k}$$

$$a \times (b \times c) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 7 & 5 \\ -36 & -36 & -24 \end{vmatrix} = 12\hat{i} - 252\hat{j} + 360\hat{k}.$$

$$\text{Again } (a \cdot c) = -71, (a \cdot b) = 49 \\ (a \cdot c)b - (a \cdot b)c = -71(-5\hat{i} + 7\hat{j} - 3\hat{k}) - 49(7\hat{i} - 5\hat{j} - 3\hat{k}) \\ = 12\hat{i} - 252\hat{j} + 360\hat{k}.$$

Thus we have

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c = 12\hat{i} - 252\hat{j} + 360\hat{k}.$$

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Product of four vector :-

A. Scalar product of four vectors

We consider the scalar product of (axb) and (cxd) i.e., we obtain
 $(axb) \cdot (cxd)$

Interchanging dot and crosses we may write
 $a \cdot b \times (cxd) = a \cdot [(b \cdot d)c - (b \cdot c)d]$
 $= (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$
we may also write this result in the form

$$\begin{vmatrix} a \cdot c & a \cdot d \\ b \cdot c & b \cdot d \end{vmatrix}$$

This form is called Lagrange's Identity.

B. vector product of four vectors

We consider the vector product of (axb) and (cxd) as $(axb) \times (cxd)$ which is a vector.

$$(axb) \times (cxd) = [abd]c - [abc]d$$

$$= [cda]b - [cdb]a$$

① If any vector d can be written as a linear combination of three non-coplanar vectors of a, b, c (i.e., $[abc] \neq 0$), then $d = \alpha a + \beta b + \gamma c$

$$\text{where } \alpha = \frac{[dbc]}{[abc]}, \beta = \frac{[dca]}{[abc]}, \gamma = \frac{[dac]}{[abc]}$$

$$\text{Thus, } d = \frac{[dbc]}{[abc]}a + \frac{[dca]}{[abc]}b + \frac{[dac]}{[abc]}c$$