12.3 Prevost's theory of exchange

Ideas regarding radiant energy were much confused prior to the *Prevost's theory of exchange* in 1792. People talked of '*cold radiations*' and '*hot radiations*' etc. For instance, it was said that a block of ice produces a sensation of cold because ice emits cold radiations. All confusions were set at naught by Prevost. According to his exchange theory, *substances at all finite temperatures emit 'radiant energy' and the amount increases with temperature and is not affected by the presence of surrounding bodies.*

When we stand near a fire, we have the sensation of warmth. Our body is also a radiator. We receive more radiant energy from the fire than what we lose by emitting our own radiation. But when we stand near a block of ice, we receive less energy from ice which is at a much lower temperature (0°C) than our body (~ 37°C), but lose more from our body by way of radiation. This gives us the feel of cold.

Once it is recognised that every substance by virtue of its temperature emits radiation, our study would be aimed at how radiation varies with *temperature* and *properties* of the body.

12.4 Instruments for detection and measurement of thermal radiation

There are several devices for the *detection and measurement* of thermal radiation. Of these, the *thermopile*, *radio-micrometer* and *bolometer* constitute the most sensitive group of instruments.

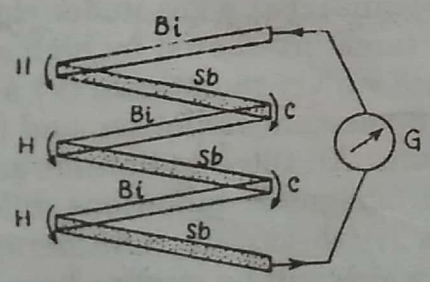

Fig.12.1. Thermopile

A. Thermopile — This is a *very sensitive* electrical device and is widely used. It consists of a large number of bars of bismuth (Bi) and antimony (Sb), placed alternately, with their ends joined such that the hot junctions of the thermocouples so formed are at one face and the cold junctions at the other. A voltage sensitive galvanometer G is included in the circuit as shown in Fig.12.1. When radiation falls upon the hot junctions, with the cold ones properly shielded, a thermocurrent flows through the galvanometer. The deflection of the galvanometer is a measure of the intensity of the incident radiation.

To *increase the sensitivity* of the device, the following measures are taken: (i) the thermoelements selected are such as to develop *a large thermo-emf*, (ii) the hot junction is coated with *lamp-black* to make it more heat-absorbing, (iii) the *junctions* are made *as small as points*, (iv) the *connecting wires* are *thin* enough to *minimise conduction* effect, (v) a *conical screen* is used to protect the *blackened hot junction from stray radiations* and (vi) the thermopile is placed *in vacuum* to eliminate the heat loss due to *convection*.

Note. The bars of bismuth and antimony are, in fact, arranged to form a cube with all the hot junction on one face and all the cold ones on the opposite face.

B. Radio-micrometer — Devised by C. V. Boys, the radiomicrometer is essentially *a thermocouple* without an external galvanometer. It consists of *two thin bars A, B* — one of bismuth and the other of antimony — joined at their lower ends to a *thin copper disc D* blackend by lampblack (Fig.12.2). The upper ends are connected to a *fine quartz fibre Q*, via a *loop C* of bare copper wire. The fibre is suspended from a thin *glass capillary G*. The loop *C* is placed between the pole pieces of a *strong permanent magnet NS*. *M* is a light *galvanometer-mirror* fixed to the glass capillary *G*. To eliminate the diamagnetic disturbance of Bi, a mass of *soft iron I* acts as a shield to the thermoelements. The whole set-up is enclosed in a brass-casing (not shown).

When radiation falls horizontally on the copper disc *D* through the *window W*, it gets absorbed and the lower junction of the thermocouple gets heated. The resulting thermocurrent passes round the circuit and as it flows through the loop *C* in the magnetic field, it experiences *a couple* deflecting it about the vertical axis. This is balanced by the *restoring couple* of the suspension fibre. The deflection of the loop is measured by the usual lamp and scale arrangement and is proportional to the intensity of the incident radiation.

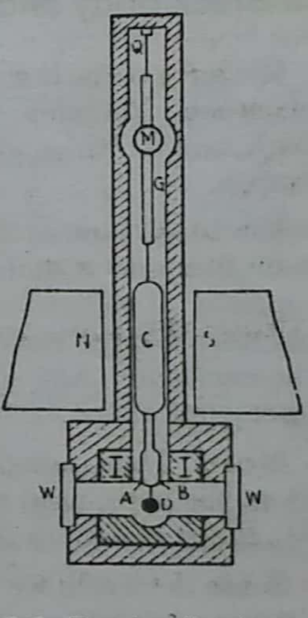

Fig.12.2. Boys' radiomicrometer

Advantages and drawbacks — The emf in a thermopile is proportional to the number of junctions and any increase in the number simultaneously increases the resistance. The radiomicrometer is free from this problem and has a number of *advantages*. It is (i) *quick in action*, (ii) *free from 'creep' of the zero* and (iii) *amazingly sensitive* so that even a very small amount of radiation can be measured. The only *drawback* it suffers from is that it can be used to receive radiation *only in the horizontal direction.*

C. Bolometer — This instrument is now seldom used being superseded by modern type of thermopiles. Devised by Langley, it depends for its action on the *change of resistance of metals* (e.g. platinum) with temperature. The sensitivity of the bolometer rests on (i) the *sensitivity of the galvanometer* and (ii) the *thinness of the strips*; thinner the strip, larger will be the rise in temperature and increase in resistance for a given amount of incident radiation.

In radiation measurements, two types of bolometers have been used : (a) *surface bolometer* for total radiation measurements and (b) *linear bolometer* for measuring the energy distribution in the spectrum of the black body.

Surface bolometer — It consists of extremely thin strips ($1 - 2 \times 10^{-3}$ cm) of platinum joined in series forming what is called a *grid*. A grid so constructed is of about 60 Ω resistance. Four such grids, S_1, S_2, S_3, S_4, identical in all respects, are connected in the form of a Wheatstone bridge (Fig.12.3). The grids 1, 3 receive the radiation that passes between the strips in 1. The effect is thus doubled. The grids 2, 4 are also similarly arranged but are properly protected from radiation by *dipping in some liquid in a thermostat.* The whole set up is encased in a box. When there is *no radiation*, the galvanometer *G* shows *no deflection*. A deflection is recorded when radiation is incident on grids 1 and 3 and the intensity of radiation is determined from the *shift in the null point.*

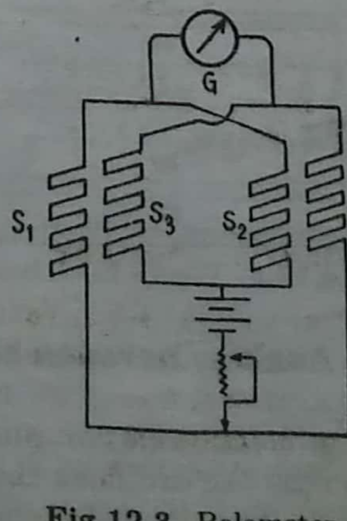

Fig.12.3. Bolometer

Linear bolometer — In the *linear type*, a single thin strip of platinum is used. One such strip is mounted in each of the two arms of the Wheatstone bridge. Abbot made a number of improvements in the construction of the bolometer to make it a very suitable instrument for use, e.g. *vacuum bolometer* that is extensively used by Astrophysical observatories.

12.5 Black body and its realisation

We have defined a *perfect black body* as one that absorbs all the radiations incident on it. A black body therefore neither transmits nor reflects any radiation. Such a body, when heated to high temperatures emits radiations of all wavelengths and such radiations are called *total radiation.*

Kirchhoff showed *theoretically* that an enclosure whose walls are impervious to any type of radiation and is maintained at a constant temperature behaves as a *perfect black body* and the quality of radiations emitted by it is that of *total radiation,* that is, they depend on the temperature of the enclosure only and are independent of the nature of the material of the walls of the enclosure. Any speck of matter when placed inside it will, *in the steady state,* attain the temperature of the enclosure and emit *black radiation* characteristic of that temperature.

Note. Thus radiation in equilibrium with matter is *black radiation.* For emitting black radiation, the body need not necessarily be black; *blackness only hastens to attain the equilibrium state.* Black radiation is also called *temperature radiation.*

Some devices have been evolved which act as perfect black bodies. They are generally of two types : one type, commonly used for *absorption experiments,* is known as *Fery's black body.* The second type, commonly used in *emission experiments,* is known as *emission black body.*

Fery's black body — Designed by Fery, it consists (Fig.12.4) of a *double walled spherical metallic shell M*, inside being *lampblacked* and the outside *F nickel-polished.* It has a *small opening* at *O* just opposite to which there is a *small conical projection C* in the inner wall. The lampblack hastens to attain the constancy of temperature and the outer polish makes the enclosure impervious to heat. The projection *C* prevents any direct reflection of rays back through the opening *O*. When any radiation enters the enclosure through *O*, it suffers multiple reflections inside and is eventually absorbed so that practically no radiation comes out through *O* again. It thus behaves as a *perfect absorber.*

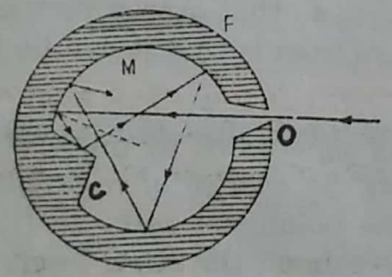

Fig.12.4. Fery's black body

Emission black body — First designed by Wien, it underwent several modifications in the hands of Lumer and Pringsheim, Coblentz and others.

It essentially consists of a *cylindrical metallic chamber C* (Fig.12.5), made of brass or platinum, *blackened inside.* The chamber is *heated electrically* by the passage of a current through *thin platinum foil H* wound over it. Concentric *porcelain tubes P, P* protect the chamber and the radiation, limited by a series of blackened concave *diaphragms D*, comes out through the opening *O*. A *thermocouple T* measures the temperature of the chamber *C*.

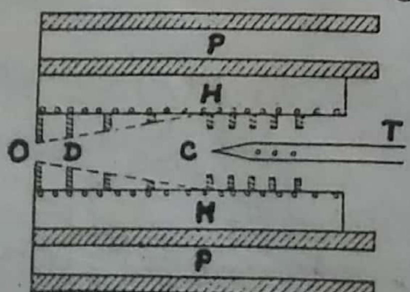

Fig.12.5. Wien's black body

12.5.1 Analogy between black radiation and an ideal gas

The *black radiation* and an *ideal gas* possess some fundamental points of *similarity* and that is why the ordinary thermodynamic relations may as well be applied to this temperature radiation. The points of similarity are :

1. An ideal gas, as pictured in the kinetic theory, is an assembly of particles in perfect chaos, that is, having all velocities from 0 to ∞, and moving in all directions. The radiations from a constant temperature enclosure (black radiation) similarly have all possible wavelengths and proceed in all directions.
2. The molecules of an ideal gas exert a pressure on the walls of the container and so does the black radiation on a surface placed normally to it.
3. The ideal gas consists of molecules having velocities, and hence kinetic energies, ranging from zero to infinity. The black radiation similarly consists of quanta or photons possessing energies varying from 0 to ∞.
4. The internal energy of an ideal gas is very similar to the energy density of radiation.

12.6 Kirchhoff's law : Some fundamental definitions

Electromagnetic radiations of all wavelenths ($\lambda = 0$ to ∞) are emitted from the surface of a heated body in all directions. The nature of the radiation depends on the physical properties of the body.

Let us first define *two* very *important quantities* that enter into the law.

Emissive power — *The emissive power e_λ of a body for the radiation of wavelength λ and $\lambda + d\lambda$ is the amount of radiation emitted per unit area of the body per second normally in unit solid angle in the direction along the axis of the solid angle.*

The emissive power of a body is a function of λ and has a *spectral character.* The total emissivity or total emissive power E is given by

$$E = \int_0^\infty e_\lambda d\lambda$$

Absorptive power — *If dQ_λ be the amount of radiant energy falling on a body in the form of radiation in the wavelength range λ and $\lambda + d\lambda$ and a fraction $a_\lambda dQ_\lambda$ of it is absorbed by the body and converted into heat, then a_λ is called the* **absorptive power** *of the body for the wavelength λ to $\lambda + d\lambda$.*

Expression for spectral emission from one side of a surface — From the definition of e_λ, it is possible to find an expression for the spectral emission from one side of a surface. The direction of axis of the solid angle is given by (θ, ϕ).

$\therefore$ Amount of radiation emitted in solid angle $d\omega$ per second from an area dA of a black body is

$$e_\lambda d\lambda dA d\omega \cos\theta$$

But $d\omega$ = solid angle between (θ and $\theta + d\theta$; ϕ and $\phi + d\phi$) $= \sin\theta d\theta d\phi$.

$\therefore$ **Spectral emission** from one side of dA

$$= dA \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} e_\lambda d\lambda \cos\theta \sin\theta d\theta d\phi = e_\lambda d\lambda dA \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta \cos\theta d\theta$$

$$= e_\lambda d\lambda dA \times 2\pi \times \tfrac{1}{2} = \pi e_\lambda d\lambda dA$$

$\therefore$ Emission per unit area per sec $= \pi e_\lambda d\lambda$.

In terms of K, *specific intensity* of radiation, i.e., the amount of radiation proceeding in a given direction per sec per unit area per unit solid angle (the entire spectrum is assumed), we can find as before the total emissivity E per unit area.

$$\therefore \quad E = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} K d\omega \cos\theta = K \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \sin\theta \cos\theta d\theta$$

$$= K \times 2\pi \times \tfrac{1}{2} = \pi K$$

Note. We may write $\pi K = \frac{1}{4}cu$, where u = energy density of radiation.

$$\therefore \quad \int_0^\infty e_\lambda d\lambda = E = \pi K = \frac{1}{4}cu = \frac{1}{4}c\int_c^\infty u_\lambda d\lambda$$

$$\Rightarrow e_\lambda = \frac{1}{4}cu_\lambda \text{ (spectral)}$$

$$\text{and} \quad E = \frac{1}{4}cu \text{ (total)}$$

Mutual radiation between two small black plates — Let dA and dA' be two small black plates, separated by r, be so oriented that the normals to the surfaces make angles θ and θ' respectively with r.

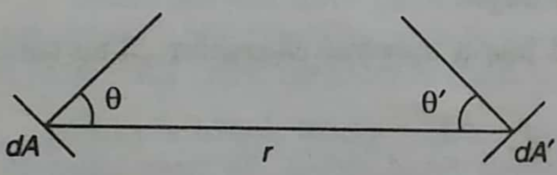

Fig.12.6

The solid angle subtended by dA' at dA is

$$d\omega' = \frac{dA' \cos\theta'}{r^2}$$

The energy emitted per unit time per unit solid angle from dA along r is $dA \cos\theta \; e_\lambda d\lambda$

$\therefore$ The energy emitted per unit time from dA and falling on dA' is

$$e_\lambda d\lambda dA \cos\theta \cdot \frac{dA' \cos\theta'}{r^2} \times 1$$

($\because$ absorptive power of black body = 1)

The expression does *not* change if dA and dA' and θ and θ' are interchanged. So the same amount of energy will also fall on dA from dA', if both the plates are at the same temperature.

Kirchhoff's law — Kirchhoff's law states that the *ratio of the emissive power to the absorptive power for radiation of a given wavelength is the same (constant) for all bodies at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.*

Symbolically,

$$\frac{e_\lambda}{a_\lambda} = \text{constant} = E_\lambda \tag{12.6.1}$$

where E_λ is the emissive power of a perfectly black body. Eq. (12.6.1) is the mathematical form of *Kirchhoff's law.*

Note 1. If a_λ fraction of energy incident on a body is absorbed, then the rest $(1 - a_\lambda)$ is reflected and/or transmitted by the body.

Note 2. The law is valid for *temperature radiation* only and does not hold for radiations obtained by chemical action or by electric discharge in gases etc.

12.6.1 Derivation of Kirchhoff's law

Consider an enclosure at a uniform temperature T, having walls *opaque* to radiations of *all wavelengths* and *insulated* thermally *from the surroundings* (Fig.12.7). The enclosure is filled with temperature radiation *emitted by the walls.* Let a body A of emissive power e_λ and absorptive power a_λ be placed inside the enclosure at a large distance from the walls. Now,

(i) Whatever be the initial temperature of the body A, it will ultimately attain the temperature T of the enclosure. For, in the equilibrium state, the entropy becomes maximum which happens only when the temperature-differences vanish.

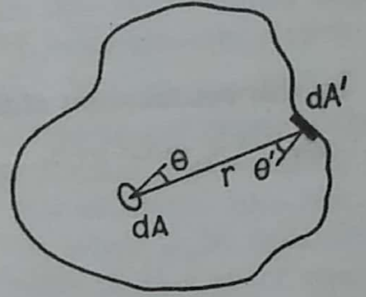

Fig.12.7. Derivation of Kirchhoff's law

(ii) Since any body placed inside finally acquires the constant temperature T of the enclosure, a heterogeneous body having different emissive and absorptive powers in different parts, if placed within the enclosure, the *total energy absorbed by it* will be *equal to the total energy it emits*, in the equilibrium state. Thus the total energy absorbed by the body is *independent of its position* or *orientation* with respect to the walls of the enclosure. As the different surfaces of the body have different absorption coefficients, this is possible when the *radiation inside* is **isotropic.**

The amount of energy emitted by an elemental area dA of the body A in the direction between $\theta, \theta + d\theta$ and $\phi, \phi + d\phi$ is

$$e_\lambda d\lambda dA \cos\theta \sin\theta d\theta d\phi$$

$\therefore$ The amount of energy emitted by dA for all wavelengths and all possible θ and ϕ is

$$dA \int_0^\infty e_\lambda d\lambda \int_0^{\pi/2} \sin\theta \cos d\theta \int_0^{2\pi} d\phi = \pi dA \int_0^\infty e_\lambda d\lambda$$

$\therefore$ For the *whole body*, the *emission* is given by

$$\pi(\Sigma dA) \int_0^\infty e_\lambda d\lambda \qquad (12.6.2)$$

But the amount of energy emitted by an elemental area dA' of the enclosure in the direction dA of the body is

$$dQ_\lambda = E_\lambda d\lambda dA' \cos\theta' \frac{dA \cos\theta}{r^2}$$

where r is the distance between dA' of the enclosure and dA of the body, E_λ the emissive power of the surface of the enclosure and θ' the angle made by the normal to dA' with the direction of emission.

$\therefore$ The amount of energy absorbed by dA of the body is given by

$$a_\lambda dQ_\lambda = a_\lambda E_\lambda d\lambda dA \cos\theta \frac{dA' \cos\theta'}{r^2}$$
$$= a_\lambda E_\lambda d\lambda dA \cos\theta d\omega = a_\lambda E_\lambda d\lambda dA \cos\theta (\sin\theta d\theta d\phi)$$

where $d\omega = dA' \cos\theta' / r^2$ = solid angle substended by dA' at $dA = \sin\theta d\theta d\phi$

$\therefore$ Total energy absorbed by dA from the total surface of enclosure for all wavelengths is

$$dA \int_0^\infty a_\lambda E_\lambda d\lambda \int_0^{\pi/2} \cos\theta \sin\theta d\theta \int_0^{2\pi} d\phi = \pi dA \int_0^\infty a_\lambda E_\lambda d\lambda$$

since the limits of integration extend for all wavelengths from 0 to ∞ and the value of the second integral is $\frac{1}{2}$ and that of the third 2π.

∴ For the *whole body*, the *absorption* is

$$\pi(\Sigma dA)\int_{0}^{\infty} a_\lambda E_\lambda d\lambda \tag{12.6.3}$$

In the *equilibrium state*, we must equate (12.6.2) and (12.6.3).

$$\therefore \quad \int_{0}^{\infty} e_\lambda d\lambda = \int_{0}^{\infty} a_\lambda E_\lambda d\lambda \tag{12.6.4}$$

Now, this equality must hold for each portion of the spectrum i.e. for any value of λ.

$$\therefore \quad e_\lambda = a_\lambda E_\lambda \tag{12.6.5}$$

If the same body A is placed in another enclosure at the same temperature T, and with E'_λ as the emissive power of its surface, but having different shape and nature of the walls,

$$e_\lambda = a_\lambda E'_\lambda \tag{12.6.6}$$

since e_λ, a_λ depend on the nature of the body and its temperature only and not upon the surroundings.

$$\therefore \quad E_\lambda = E'_\lambda$$

If a black body, that is, having $a_\lambda = 1$ be placed inside the enclosure, we have from (12.6.5), $E_\lambda = e_\lambda$; that is E_λ is equal to the *emissive power* of a *black body*. Thus, *radiation in any hollow enclosure is independent of the nature and shape of the walls and is identical with black body radiation at the same temperature.*

Kirchhoff's law now immediately follows. From (12.6.5), we have $e_\lambda/a_\lambda = E_\lambda$, that is, *at any temperature, the ratio of the emissive to the absorptive power of a substance is a constant,* being *equal to the emissive power of a perfectly black body.* This is **Kirchhoff's law.**

12.6.2 Importance of Kirchhoff's law : Applications

The importance of Kirchhoff's law can hardly be overestimated. The law has two distinct aspects : (i) *qualitative* and (ii) *quantitative. Qualitatively* it says that if a body is capable to emit certain radiations when excited, it will absorb them when they are incident on it; *quantitatively,* it signifies that the ratio of the emissive to the absorptive power is the same for all bodies.

Various observations support the qualitative relation. For instance, if a polished metallic ball with a black spot (Pt-black coat) is heated to about 1000°C and suddenly taken out in a dark room, the black spot shines more brilliantly than the polished surface. This is because, the black being a better absorber, at emits also much greater light. This *selective action* is well demonstrated by oxides of metals like erbium which emit, on heating, certain bands in addition to the continuous spectrum. If continuous light is passed through a solution of these oxides, the same very *bands appear in absorption.*

But the importance of Kirchhoff's law is that it has, in a sense, opened up two entirely new branches: (i) **astrophysics** and (ii) **spectroscopy**. Newton demonstrated by passing sunlight through a prism that it consists of a spectrum of seven colours. More than a hundred year later, Fraunhofer in repeating Newton's experiment found that the solar spectrum is not continuous but is crossed by a large number of dark lines. He noticed 500 of them (now about 20,000) and designated the prominent ones by the letters $A, B, C, D, \cdots$. But neither Fraunhofer nor his contemporaries could understand how the *Fraunhofer lines* originate. A similar effect was also observed in the stellar spectrum. Subsequently, physicists like Fizeau observed that if the solar spectrum is examined side by side with that of a Na-flame, the two yellow lines appear exactly at the place of D-band of Fraunhofer. Similar is the case with hydrogen spectrum.

The explanation given by Kirchhoff *completely solved* the problem. He supposed that the central core (*photosphere*) of the sun is a glowing mass that emits a continuous spectrum without the dark lines. But this light has to pass through a *relatively cooler* mantle surrounding the core (*chromosphere*). The mantle however is hot enough to have all the elements like Na, Cu etc. in it in the gaseous form. Sodium emits D lines when excited. So when light falls on it, it will absorb the same light and let others pass through. Thus the gases in the outer mantle deprive the continuous spectrum of the central core of those particular lines they themselves can emit, and give rise to the *black lines*.

The flash-spectrum further confirms the correctness of Kirchhoff's explanation. We supposed that the outer solar atmosphere has Na and so if we could observe the spectra of the outer atmosphere, excluding the central core, the Na-lines would appear bright. Now the exclusion of the central core is possible only during a total solar eclipse. So during total eclipse when the outer atmosphere is laid bare, the dark lines will flash out as bright lines which was indeed observed by Young in 1872.

But Kirchhoff's discovery is of still far-reaching consequence. It asserts that *every different type of atom, when properly excited, emits light of a different wavelength characteristic of the atom.* So each atom can be identified by the particular line it emits. Thus was born the branch **Spectrum analysis** that aims at identifying elements by their characteristic lines. It is thrilling to note that as many as 40 new elements were thus added to the list already known.

12.7 Pressure of radiation

Radiation being identical with light, it is expected that it must exert a *small* but *finite pressure* on surfaces on which it is incident. In fact, the idea on the existence of radiation pressure came first from the observations of Kepler that the tail of comets, as they approach the sun, continuously veers round so as to be always opposite to the sun. A *theoretical footing* to this idea was provided by the electromagnetic theory of Maxwell. The quantum theory of light explains it more directly and also gives an expression for it.

From quantum theory — According to the quantum theory, radiation consists of *photons*, each having *energy content* $h\nu$ and moving with velocity c, ν being the frequency of the photon. Let us consider such a photon of energy $h\nu$. By the *mass-energy relation* of Einstein, energy $E = mc^2$.

$$\therefore \quad h\nu = E = mc^2$$

$$\therefore \quad \text{mass of the photon, } m = h\nu/c^2$$

$$\therefore \quad \text{momentum of the photon} = mc = h\nu/c$$

If this photon is incident normally on a black surface and gets absorbed, it will exert an impulse $= h\nu/c$.

$\therefore$ Total pressure exerted on the surface, $p = \Sigma h\nu/c$ where the summation extends over photons of all frequencies incident per second on the surface.

$\therefore$ **Radiation pressure**, $p = \Sigma h\nu/c = I/c$ where $I = \Sigma h\nu =$ intensity of light.

Bartoli's proof — An interesting *thermodynamic proof* of the existence of radiation pressure is due to Bartoli.

He imagined a cylinder $DABC$ (Fig.12.8) with *perfectly reflecting surface* and closed at both ends by two *perfectly conducting black plates* C, D in contact with two heat-reservoirs at temperatures $T, T'(T' > T)$. A and B are two *perfectly reflecting diaphragms*, B being provided with a *valve* V.

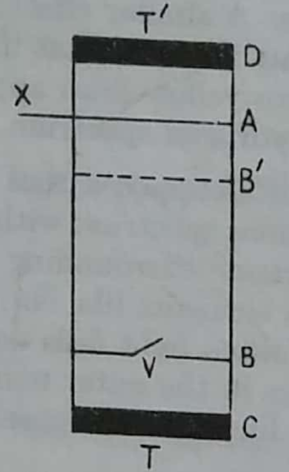

Fig.12.8. Bartoli's proof of radiation pressure

First, the valve V is open, A being in the position as shown so that space AB is filled with radiation in equilibrium with a black body at T, and space AD with radiation in equilibrium with a black body at T'. V is next closed and B moved upward to B' compressing the radiation trapped in AB to a very small volume. The density of radiation in AB' is thus much higher than that in AD. If now A is taken out, the space $B'D$ will contain more radiation than what can remain in equilibrium with black body at T'. The excess energy will be absorbed by the black body at T' till equilibrium is restored. The net effect of the process would thus be a *transference of heat* from a *colder to a hotter* body which, by the second law, is possible only by expenditure of *work*. So some work has been done in moving B upwards. This implies that B is moved against a pressure — the *radiation pressure*.

12.8 Energy density and pressure of diffuse radiation

In the case of *diffuse radiation*. e.g. the radiation inside a heated enclosure, the relation is not that simple as $p = I/c$. Let us now find an expression for the energy density of radiation inside a uniformly heated enclosure of any shape at a certain temperature.

Energy density — Let v be a small elemental volume inside the enclosure and at a large distance from the walls. All the radiations inside v may be supposed to come from the surface of a sphere described about any point O inside v, the radius r of the *sphere* is very large compared to the dimensions of v (Fig.12.9a).

Let the radiation from an elemental surface dA of the enclosure traverse along the radius to intersect v in a length l. With a point on dA as vertex and the radial line as axis, an elementary *cone* is described, the *solid angle* of which is given by $d\omega = \alpha/r^2, \alpha$ being the area of cross-section *perpendicular* to the radius. The time taken by the radiation to traverse the volume v is $t = l/c$, where c is the velocity of radiation.

$\therefore$ The amount of radiation contained in v due to the elementary cone under reference is

$$K dA \frac{\alpha}{r^2} \cdot \frac{l}{c} \tag{12.8.1}$$

since the inclination $\theta \simeq 0, K$ being the *specific intensity of radiation*, i.e. the amount of radiation proceeding in a given direction per sec per unit area per unit solid angle.

$\therefore$ The total amount of radiation contained in v due to dA is

$$\sum K dA \frac{\alpha}{r^2} \frac{l}{c} = \frac{K dA}{r^2 c} \sum \alpha l = \frac{K dA}{r^2 c} v \quad (\because \ \Sigma \alpha l = v) \tag{12.8.2}$$

obtained by summing the expression (12.8.1) over all the elementary cones.

Summing the expression (12.8.2) again over all elements dA of the sphere, the total energy of radiation in v due to the entire enclosure is

$$\sum \frac{K dA}{r^2 c} v = \frac{Kv}{c} \sum \frac{dA}{r^2} = \frac{Kv}{c} 4\pi$$

$\therefore$ Energy density of diffuse radiation, $u = \dfrac{4\pi K}{c}$.

Pressure and energy density — Consider a surface AB (Fig.12.9b) on which radiation is incident in a direction θ with the normal to AB. The intensity of radiation in the enclosure due to the radiation coming from the direction enclosed in a small solid angle $d\omega$ is $K d\omega$ and may be treated as *linear*.

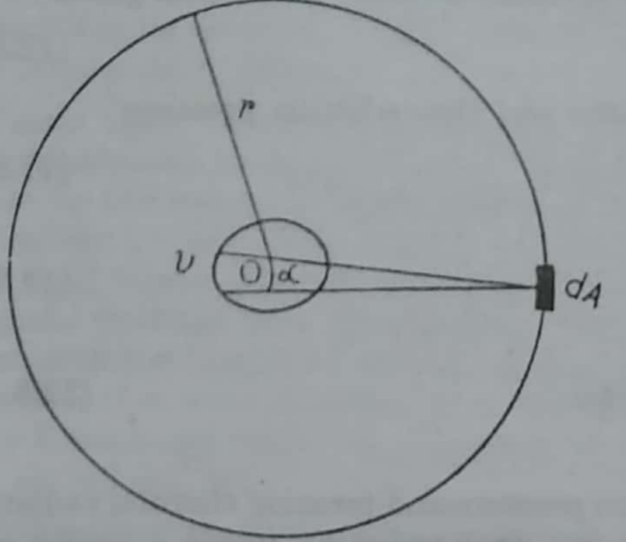

Fig.12.9a. Energy density of diffuse radiation

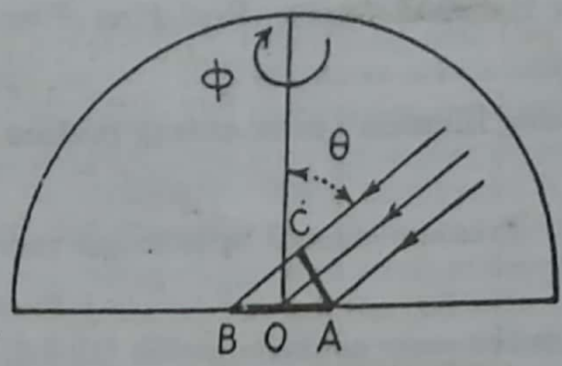

Fig.12.9b. Diffuse radiation incident on the surface AB

$\therefore$ Pressure on AC due to radiation from (θ, ϕ) is $Kd\omega/c$, where $d\omega = \sin\theta d\theta d\phi$.

$\therefore$ Force on $AC = (Kd\omega/c).AC$ = Force on AB.

The normal component of the above force is : $(K/c)d\omega \cdot AC\cos\theta$.

$\therefore$ Pressure on AB due to radiation incident normally on it from (θ, ϕ) is

$$\frac{K}{c}d\omega\frac{AC\cos\theta}{AB} = \frac{K}{c}\cos^2\theta d\omega \tag{12.8.3}$$

Integrating (12.8.3) over the entire hemisphere, we obtain the total pressure p.

$$\therefore \quad p = \int_0^{\pi/2}\int_0^{2\pi}\frac{K}{c}\cos^2\theta\sin\theta d\theta d\phi = \frac{K}{c}\int_0^{\pi/2}\cos^2\theta\sin d\theta\int_0^{2\pi}d\phi = \frac{K}{c}\cdot\frac{1}{3}2\pi \tag{12.8.4}$$

But the energy density u, in this case, is given by $u = 2\pi K/c$.

$$\therefore \quad p = \tfrac{1}{3}u \tag{12.8.5}$$

12.8.1 Alternative treatment : Kinetic theory approach

It is possible to derive the expression for pressure exerted by thermal radiation (12.8.5) by considering it as a *photon gas* from kinetic-theory standpoint.

The radiation is assumed to be *isotropic* (that is, *identical* in quality and quantity *in all directions*) and trapped in a cavity with perfectly reflecting walls. The photons, of which the radiation consists, move in random directions within the cavity and rebound elastically from its walls.

The situation is analogous to that of an ideal gas contained in a similar vessel. From kinetic theory, the number of molecules striking unit area of wall per second is

$$N = \tfrac{1}{4}n\bar{c}$$

n being the number of molecules per unit volume and $\bar{c}$, the mean molecular speed. In case of photon gas, all the photons move with the same speed c, the speed of light; so the number of photons striking unit area per second is

$$N = \tfrac{1}{4}nc$$

If the average energy of photon is $\bar{\epsilon}$, the radiant power on the wall per unit area is

$$P = \tfrac{1}{4}nc\bar{\epsilon} = \tfrac{1}{4}(n\bar{\epsilon})c = \tfrac{1}{4}uc \tag{12.8.6}$$

where u is the *energy density* of radiation.

Pressure due to isotropic radiation — The pressure exerted by an ideal gas is

$$p = \frac{1}{3}\rho \bar{c^2} \tag{12.8.7}$$

ρ being the mass-density. Replacing $\bar{c^2}$ by c^2 for photon gas, the radiation pressure

$$p_{rad} = \frac{1}{3}\rho c^2 \tag{12.8.8}$$

Using Einstein's mass-energy relation :

$$u = \rho c^2 \tag{12.8.9}$$

$\therefore$ Pressure exerted by isotropic radiation

$$p_{rad} = \frac{1}{3}\rho c^2 = \frac{1}{3}u \tag{12.8.10}$$

which is the same as the equation (12.8.5).

Note. Using the expression (12.8.10) for radiation pressure and treating thermal radiation as a thermodynamic system (justified owing to the fact that radiation inside a cavity is a radiation in thermodynamic equilibrium with matter as shown by Kirchhoff), we obtain, at a temperature T,

$$u = \alpha T^4 \tag{12.8.11}$$

where α is a constant.

From (12.8.6) and (12.8.11), we get $P = \frac{1}{4}\alpha c T^4 = \sigma T^4$, which gives the Stefan-Boltzmann law (discussed later in Art.12.10) in terms of radiated energy emitted per second per unit area by a black body. The $\sigma\ (= \frac{1}{4}\alpha c)$ is known as *Stefan constant.*

12.9 Experimental proof of radiation pressure

The existence of radiation pressure was successfully demonstrated by Lebedew in 1900 and a little later by Nichols and Hull.

Nichols-Hull arrangement — The arrangement essentially consists of a *pair of vanes* A, A' of absorbing (blackened) or reflecting (polished) surfaces, mounted on the arms of a *glass cross*, and suspended by a means of a thin *quartz fibre* F inside a *bell jar* (Fig.12.10). To the suspension is attached a *mirror* M and the deflection of the suspended system is measured by the usual *lamp and scale* arrangement. Light from a source, rather strong, is focussed on one of the vanes, say A, the blackened vane and the resulting deflection measured. The temperature of the blackened face that absorbs the radiation is raised above the temperature of the polished face and this causes the deflection θ.

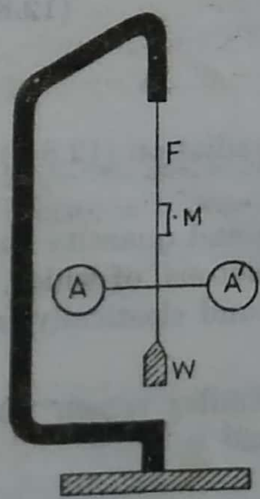

Fig.12.10. Apparatus for measuring radiation pressure

If α be the effective area of the vanes, d the deflection of light spot, D the distance between the mirror and the scale, l the arm length of the vanes and p the pressure of radiation,

$$p\alpha l = c\theta = c\frac{d}{2D}$$

where c is the torsional constant of the quartz fibre.

$$\therefore \quad p = \frac{c}{\alpha l}\frac{d}{2D}$$

The *disturbing factors* in such experiments are the following.

(i) *Convection current:* Convection current of air is set up due to the rush of air towards the parts warmed up due to the passage of the pencil of radiation.

(ii) *Rocket action:* This is due to the escape of gas particles from the surface of the vanes when they are heated by incident radiation. It results in a recoil experienced by the vanes.

(iii) *Radiometric action:* This action is due to unequal heating of the two sides of the vane and results in its rotation. This is proportional to the pressure of the surrounding gas and also to the temperature difference. In general, it is greater than the true radiation pressure.

Unless these effects are eliminated, the success is impossible. Factors (i) and (iii) are usually eliminated by suspending the vanes in the *highest vacuum* and taking as *thin* a *strip* as possible for the vanes. Lebedew used very thin strips of platinum and carried out experiments at extremely low pressures. Nichols and Hull, on the other hand, used a rather higher pressure ($\sim$ 16 mm) where the radiometric action is found to be *minimum.* They eliminated it by differential readings from the two sides. Hull later placed the silver side of a thin cover glass in contact with the blackened side of a similar glass, the whole being enclosed in a cell of two thin glass plates that were separated from these by air space. Both the surfaces of the cell were thus equally heated and there was practically *no* radiometric action. Factor (ii) was also eliminated as *no gas left* the cell.

Saha-Chakravarti's arrangement — Saha and Chakravarti in 1918 devised a simple arrangement on similar lines. We quote below Professor Saha himself.

The silvered sides of two thin circular glass slides (as used in microscopical studies) were put one upon the other and connected to each other by a trace of canada-balsam on the fringes, thus enclosing a thin film of air in between. When light, previously filtered of all rays capable of heating glass, is allowed to fall on one side of this vane, the two sides of the film are instantly raised to the same temperature (because it is extremely thin) and hence the *radiometric action* is eliminated. Two such vanes were suspended on the opposite arms of a torsional balance which is suspended (Fig.12.10) by a thin quartz fibre F having a mirror M inside a vessel that can be evacuated. On account of the low pressure *convection currents* are absent. The *rocket action* is also eliminated since the liberated gas remains inside the system, and the action and reaction being equal, the net effect is zero.