14.04.20

RADIATION

① Reflection coefficient :— Reflection coefficient is the fraction of the incident radiation reflected by the body.

i.e. $r = \dfrac{\text{Amount of radiation reflected by the body}}{\text{Amount of radiation incident on the body}}$

② Absorption coefficient :— Absorption coefficient is the fraction of the incident radiation absorbed by the body.

i.e. $a = \dfrac{\text{Amount of radiation absorbed by the body}}{\text{Amount of radiation incident on the body}}$

③ Transmission coefficient :— Transmission coefficient is the fraction of the incident radiation transmitted by the body.

i.e. $t = \dfrac{\text{Amount of radiation transmitted by the body}}{\text{Amount of radiation incident on the body}}$

N.B — ①. $r+a+t=1$

②. If $r=1$ and $a=t=0 \Rightarrow$ called perfect white body

③ If $a=1$ and $r=t=0 \Rightarrow$ called perfect black body

④ If $t=1$ and $r=a=0 \Rightarrow$ called perfect transparent body

④ Define black body and what is its realisation ?

Perfect blackbody :— A perfect blackbody is one that absorbs all the radiations incident on it, neither reflects or transmits any radiation.

Such a body, when heated to high temperatures emits radiations of all wavelengths and such radiations are called total radiation.

Realisation of perfect black body ——

Kirchhoff showed theoretically that an enclosure whose walls are impervious of to any type of radiation and is maintained at a constant tempr. behaves like a perfect blackbody. Total radiation depend on the tempr. of the enclosure only and are independent of the nature of the materials of the walls of the enclosure.

(i) Fery's black body :——

Define emissive power and absorptive power :—

(1) Emissive power :—

The emissive power e_λ of a body for the radiation is range λ and $\lambda+d\lambda$ is the amount of radiation emitted per unit area of the body per second normally in unit solid angle in the direction along the axis of the solid angle.

$$\text{i.e. } e_\lambda = \frac{u_\lambda}{dA \cdot dt \cdot \cos\theta \, d\omega}$$

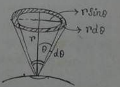
Where amount of radiation u_λ emitted from a body through an area dA. and solid angle $d\omega$ in time dt at an angle θ with the axis of the solid angle.

(2) Absorptive power :—

Absorptive power is the fraction of radiation absorbed by the body. If dQ_λ be the amount of radiation energy falling on a body in the wavelength range λ and $\lambda+d\lambda$ and a fraction dQ_λ' is absorbed by the body then the absorptive power of the body,

$$a_\lambda = \frac{dQ_\lambda'}{dQ_\lambda}.$$

* Idea about solid angle —



i) Solid angle $d\omega = \dfrac{\overset{\text{perimeter}}{\text{area}} \times \overset{\text{Thickness}}{\text{depth}}}{r^2}$.

$$= \frac{2\pi r\sin\theta \cdot r\,d\theta}{r^2}$$

$$= 2\pi \sin\theta \, d\theta$$

ii)

$$d\omega = \frac{dA\cos\theta}{r^2}$$

State and proof Kirchhoff's law of radiation

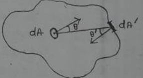
Kirchhoff's law (statement) — It sates that, at any tempr. and for the same wavelength, the ratio of the emissive power to the absorptive power for all bodies is constant and is equal to the emissive power of a perfect black body at that tempr.

Symbolically, $\frac{e_\lambda}{a_\lambda} = \text{constant} = E_\lambda$

Where E_λ is the emissive power of a perfect black body.

Derivation —

We consider a hollow enclosure at a uniform tempr. T, having walls opaque to radiations of all wavelengths and insulated thermally from the surroundings. Let a body A of emissive power e_λ and absorptive power a_λ be placed inside the enclosure at a large distance from the walls.



Now radiations will fall on the body from all sides of the walls of the enclosure and the body soon attains the tempr. of the enclosure i.e. equilibrium state is established. Again in eqm state, the total energy absorbed by the body will be equal to the total energy it emits.

(i) The amount of energy emitted by an elementary area dA of the body A in an angle θ and θ+dθ which falls on dA' for wavelength λ and λ+dλ

$= e_\lambda d\lambda \, dA \cos\theta \times d\omega'$

$= e_\lambda d\lambda \, dA \cos\theta \times 2\pi \sin\theta \, d\theta$

The amount of energy emitted by dA for all wavelengths and all possible θ

$$= 2\pi dA \int_0^\infty e_\lambda d\lambda \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$$

$$= 2\pi \, dA \times \frac{1}{2} \cdot \int_0^\infty e_\lambda d\lambda$$

$$\int_0^{\pi/2} \sin\theta \cos\theta \, d\theta = \frac{1}{2}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \frac{1}{2}$$

For the whole body, the emission is given

$$\text{by } Q = \sum \pi dA \int_0^\infty e_\lambda d\lambda \quad \text{——} (1)$$

(ii) The amount of energy emitted by an elementary area dA' of the enclosure in the direction dA of the body

$$= E_\lambda d\lambda \, dA' \cos\theta' \, d\omega$$

$$= E_\lambda d\lambda \, dA' \cos\theta' \cdot \frac{dA \cos\theta}{r^2}$$

$$= E_\lambda d\lambda \, dA \cos\theta \, \frac{dA' \cos\theta'}{r^2}$$

$$= E_\lambda d\lambda \, dA \cos\theta \cdot \frac{dA' \cos\theta'}{r^2}$$

$$= E_\lambda d\lambda \, dA \cos\theta \cdot d\omega'$$

$$= E_\lambda d\lambda \, dA \cos\theta \times 2\pi \sin\theta \, d\theta$$

Where E_λ is the emissive power of the surface of the enclosure.

The amount of energy absorbed by dA of the body is,

$$= a_\lambda \times E_\lambda d\lambda \, dA \cos\theta \times 2\pi \sin\theta \, d\theta$$

∴ Total amount of energy absorbed by dA from total surface of enclosure for all wavelengths

$$= 2\pi dA \int_0^\infty a_\lambda E_\lambda d\lambda \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$

$$= \pi dA \int_0^\infty a_\lambda E_\lambda d\lambda$$

For the whole body the absorption is given

$$\text{by } Q' = \sum \pi dA \int_0^\infty a_\lambda E_\lambda d\lambda \quad \text{——} (2)$$

In the equilibrium state, $Q = Q'$

$$\Rightarrow \int_0^\infty e_\lambda \, d\lambda = \int_0^\infty a_\lambda E_\lambda \, d\lambda .$$

The above equality holds for any value of λ,

Thus $e_\lambda = a_\lambda E_\lambda$.

$$\therefore \boxed{\frac{e_\lambda}{a_\lambda} = E_\lambda} \quad \underline{\text{Proved}}$$

If a blackbody, having $a_\lambda = 1$ be placed inside the enclosure then $e_\lambda = E_\lambda$ then emissive power of the black body $(e_\lambda) = E_\lambda$.

Importance of Kirchhoff's law :—

The importance of Kirchhoff's law is —

- i) It shows that good emitters are also good absorbers i.e, if a body emits certain radiation then it will also absorb under suitable circumstances.
- ii) A hollow enclosure at a uniform temperature behaves like a perfect blackbody.

Application of Kirchhoff's law of radiation :—

Kirchhoff's law opened up two entirely new branches —

i) astrophysics.

ii) spectroscopy.

(i) $\boxed{\text{Astrophysics application}}$ — Fizeau observed that if the solar spectrum is examined side by side with that of Na-flame, the two yellow lines ($\lambda_1 = 5890\,A^\circ$ and $\lambda_2 = 5896\,A^\circ$) appear exactly at the place of D-band of Fraunhofer.

* (In Newton's experiment Fraunhofer found that solar spectrum is not continuous but is crossed by a large number of dark lines. These lines are called Fraunhofer lines.)

The above problem was explained by Kirchhoff's law as — The central core i.e photosphere of the sun emits a continuous spectrum without the dark lines. But this light has to pass through relatively cooler surrounding chromosphere which contains Na, Cu etc. in the gaseous form. This Na absorb. the D_1 and D_2 yellow lines.

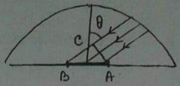
Thus when solar spectrum is examined by Na-f flame then it will emits that two lines.

Spectroscopy?.

(ii) Astrophysic's application — Kirchhoff's law asserts that every different type of atom, when properly excited, emits light of different wavelength characteristic of the atom. So each atom can be identified by the particular line it emits. Thus 40 new b elements were added to the list already known.

Pressure of diffuse radiation. (4 marks)

Consider a surface AB on which a parallel beam of radiation is incident in a direction θ with the normal to the surface AB.



The intensity of radiation in the enclosure in a solid angle $d\omega$ is $I = K d\omega$

Pressure on AC due to radiation in solid angle $d\omega$.

$$P = \frac{I}{c} = \frac{K d\omega}{c} = \frac{K\, 2\pi \sin\theta\, d\theta}{c}$$

$\therefore$ Force on AC $= \frac{2\pi K \sin\theta\, d\theta}{c} \times AC$ = Force on AB.

Normal component of above force on AB

$$= \frac{2\pi K \sin\theta\, d\theta}{c} \times AC \times \cos\theta$$

$$= \frac{2\pi K}{c} \times AB \times \sin\theta \cos\theta\, d\theta$$

Total pressure exerted on AB due to diffuse radiation from two sides,

$$P = \frac{4\pi K}{c} \int_0^{\pi/2} \sin\theta \cos\theta\, d\theta \cdot \frac{AB}{AB}$$

$$= \frac{4\pi K}{c} \int_0^{\pi/2} \sin\theta \cos^2\theta\, d\theta \qquad \text{as } AB = AB\cos\theta$$

$$= \frac{4\pi K}{c} \times \frac{1}{3}$$

$\therefore \boxed{P = \frac{1}{3} u}$ as energy density $u = \frac{4\pi K}{c}$.

V.U.01 State and derive Stefan-Boltzmann law :—

Stefan's law (Statement) — Total energy of radiation emitted by a blackbody/sec is proportional to the fourth power of its absolute temperature

i.e. $\boxed{Q \propto T^4}$

Stefan-Boltzmann law (Statement) —

If a blackbody at absolute temperature T be surrounded by another blackbody at absolute temperature T_0, then amount of radiation emitted per second per unit area of the former, is.

$$Q = \sigma\left(T^4 - T_0^4\right)$$

where σ is the Stefan constant.

$\sigma = 5.6 \times 10^{-8}$ watt/m^2/K^4

Derivation — (Applying Thermodynamics)

Let u be the energy density of radiation in an enclosure at constant absolute temp T.

Volume of the enclosure = V

Pressure of radiation = p.

∴ Total energy of radiation in the enclosure $U = uV$

From 1st law of thermodynamics,

$$\begin{aligned} dQ &= dU + pdV \\ &= d(uV) + pdV \\ &= udV + Vdu + pdV \\ &= udV + Vdu + \tfrac{1}{3}udV \\ &= Vdu + \tfrac{4}{3}udV \end{aligned}$$

as $p = \frac{1}{3}u$

$\therefore \quad TdS = Vdu + \frac{4}{3}udV$. From second law, $dQ = T\,dS$

$$\Rightarrow dS = \frac{V}{T}du + \frac{4u}{3T}dV \quad \text{——} (1)$$

$$= M\,du + N\,dV$$

Since ds is a perfect differential,

$$\frac{\partial}{\partial V}(M) = \frac{\partial}{\partial u}(N)$$

$$\Rightarrow \frac{\partial}{\partial V}\left(\frac{V}{T}\right)_u = \frac{\partial}{\partial u}\left(\frac{4u}{3T}\right)_V$$

$$\Rightarrow \frac{1}{T} = \frac{4}{3}\frac{1}{T} - \frac{4}{3}\frac{u}{T^2}\cdot\frac{dT}{du}$$

$$\Rightarrow \frac{4}{3}\frac{u}{T}\cdot\frac{dT}{du} = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\Rightarrow 4\frac{dT}{T} = \frac{du}{u}$$

Integrating, $\int \frac{du}{u} = 4\int\frac{dT}{T}$

$$\Rightarrow \ln u = 4\ln T + \ln\alpha$$

$$\Rightarrow \boxed{u = \alpha T^4}$$ Where $\alpha \Rightarrow$ ~~total~~ int. const.

Again radiation lost per second per unit area i.e Q is proportional to energy density.

i.e $Q \propto u$

$\therefore Q \propto \alpha T^4$

$$\boxed{Q = \sigma T^4}$$ Where σ = Stefan's const.

When this body is surrounded by another body at temp^r T_0 then ~~it~~ the latter will also emit radiation by an amount σT_0^4 which will incident to the inner body.

$\therefore$ Thus the net loss of radiation of the first body per second ~~per second~~ unit area would be,

$$\boxed{Q = \sigma(T^4 - T_0^4)}$$

Problem
V.U - '94 (Zb)
V.U - 99, 2000
V.U - 98

Prove that for a reversible adiabatic change of volume V of the enclosure, $VT^3 = \text{constant}$, Where T is temp^r of the black body in absolute scale.

Solution —

From 1st law of thermodynamics,

$$dQ = dU + pdV$$

$$\Rightarrow dQ = d(uV) + pdV$$

Total energy $U = uV$

$u \Rightarrow$ energy density

$V \Rightarrow$ volume

P = pressure of radiation $= \frac{1}{3}u$

$$\Rightarrow dQ = Vdu + udV + \frac{1}{3}udV$$

$$\Rightarrow dQ = Vdu + \frac{4}{3}udV$$

For adiabatic process, $dQ = 0$

$$\therefore \quad Vdu + \frac{4}{3}udV = 0$$

$$\Rightarrow \quad \frac{du}{u} + \frac{4}{3}\frac{dV}{V} = 0$$

Integrating, $\int \frac{du}{u} + \frac{4}{3}\int \frac{dV}{V} = \int 0$

$$\Rightarrow \ln u + \frac{4}{3}\ln V = \ln C$$

$$\Rightarrow \quad uV^{4/3} = C \quad \text{——} (1)$$

Again, energy density $u = \sigma T^4$ — (2), where σ = Stefan's constant

From equation (1) and (2), we can write,

$$(\sigma T^4)(V)^{4/3} = C$$

$$\Rightarrow \quad VT^3 = \frac{C}{\sigma^{4/3}} = \text{const}$$

$$\therefore \boxed{VT^3 = \text{constant}} \quad \text{Proved}$$

State and prove derive Newton's law of cooling :—

Newton's law of cooling —— Heat loss per second due to radiation is directly proportional to the temp. difference between the body and the surroundings when this difference is small.