Semester-II
B.Sc (Honours) in Physics

CC3T: Electricity and Magnetism-C3

Lecture
on
Network theorems

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Lecture-I

(Specially: Ideal Constant-voltage and Constant-current Sources)

Syllabus

Network theorems

- **Ideal Constant-voltage and Constant-current Sources.**
- **Network Theorems:**
  - **Thevenin theorem,**
  - **Norton theorem,**
  - **Superposition theorem,**
  - **Reciprocity theorem,**
  - **Maximum Power Transfer theorem.**
- **Applications to dc circuits**

Contains

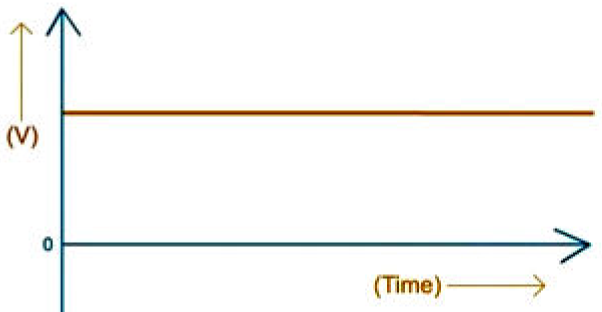
- Direct Current (DC)
- Alternating Current (AC)
- Voltage Source
  - Direct voltage source
  - Alternating voltage source
- Constant Voltage Source
- Constant Current Source
- Conversion of Voltage Source into Current Source
- Problems

Direct Current (DC)

Direct Current (DC), which is a constant stream of electrons in one direction. DC provides a constant voltage or current.

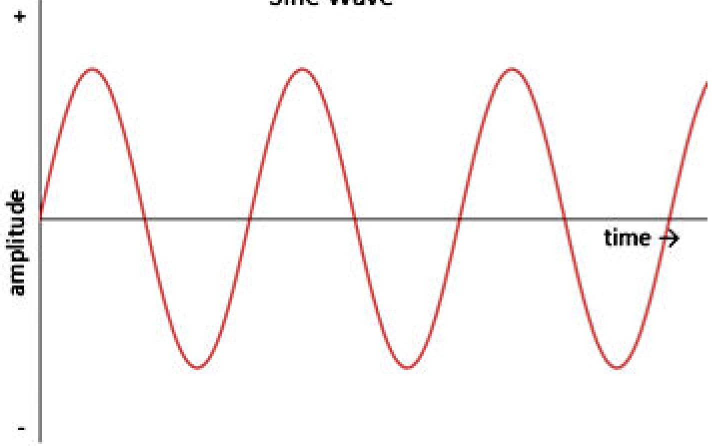
Alternating Current (AC)

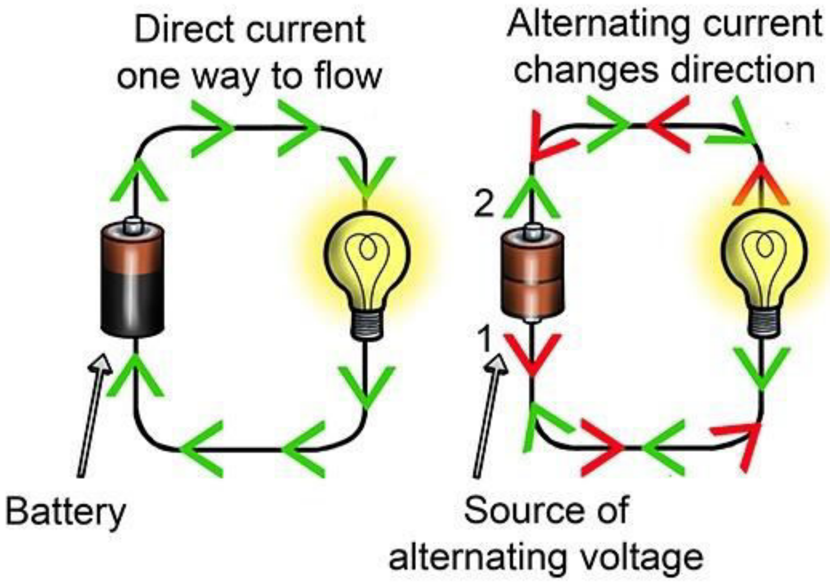
Alternating current describes the flow of charge that changes direction periodically. As a result, the voltage level also reverses along with the current. AC is used to deliver power to our households(which we normally get in our houses) and industries.



Direct Current Graph

Sine Wave





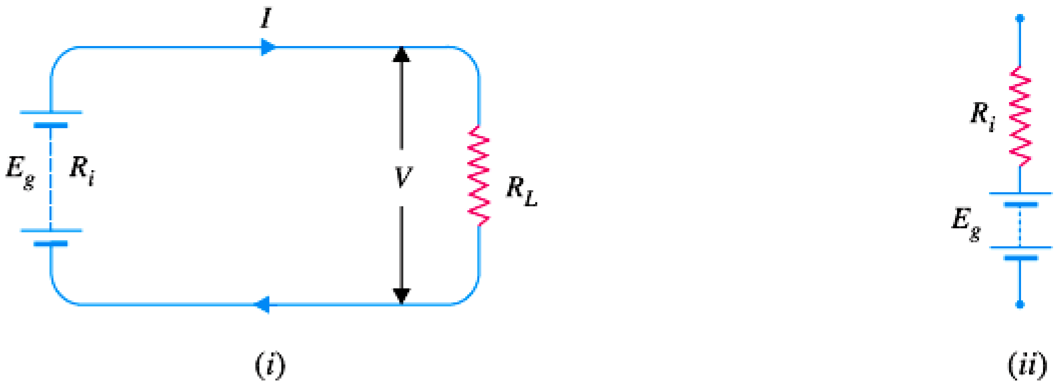
Voltage Source

Any device that produces voltage output continuously is known as a **voltage source**. There are two types of voltage sources, namely ; **direct voltage source** and **alternating voltage source.**

(i) **Direct voltage source**. A device which produces direct voltage output continuously is called a direct voltage source. Common examples are cells and d.c. generators. An important characteristic of a direct voltage source is that it maintains the **same polarity of the output voltage** i.e. positive and negative terminals remain the same.

When load resistance R_L is connected across such a source, current flows from positive terminal to negative terminal via the load [See Fig. 1(i)]. This is called direct current because it has just one direction. The current has one direction as the source maintains the same polarity of output voltage.

The opposition to load current inside the d.c. source is known as internal resistance Ri. The equivalent circuit of a d.c. source is the generated e.m.f. Eg in series with internal resistance Ri of the source as shown in Fig. 1 (ii). Referring to Fig. 1 (i), it is clear that:



(i) (ii)

Fig. 1

$$\text{Load current, } I = \frac{E_g}{R_L + R_i}$$

$$\text{Terminal voltage, } V = (E_g - I\,R_i) \quad \text{or} \quad I\,R_L$$

(ii) **Alternating voltage source**. A device which produces alternating voltage output continuously is known as alternating voltage source e.g. a.c. generator. An important characteristic of alternating voltage source is that it periodically reverses the **polarity of the output voltage**. When load impedance Z_L is connected across such a source, current flows through the circuit that periodically reverses in direction. This is called alternating current.

The opposition to load current inside the a.c. source is called its internal impedance Z_i. The equivalent circuit of an a.c. source is the generated e.m.f. E_g (r.m.s.) in series with internal impedance Z_i of the source as shown in Fig. 2 (ii). Referring to Fig. 2 (i), it is clear that :

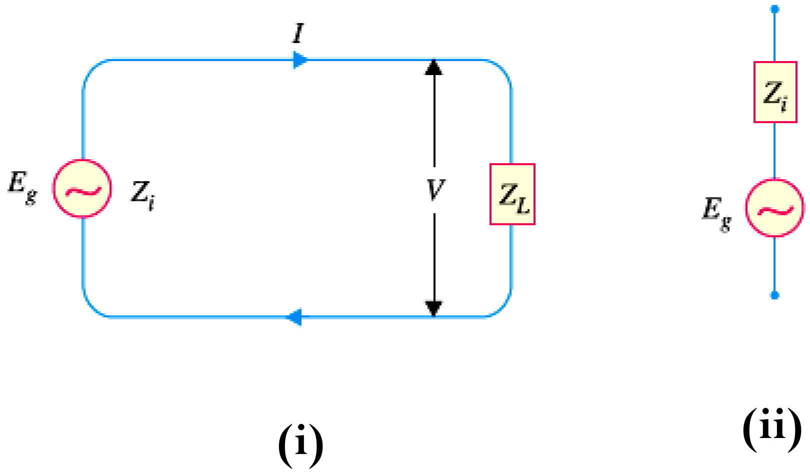


Fig. 2

$$\text{Load current, } I\,(r.m.s.) = \frac{E_g}{Z_L + Z_i}$$

$$\text{Terminal voltage, } V = (E_g - I\,Z_i)^{**} \quad \text{or} \quad I\,Z_L$$

Constant Voltage Source

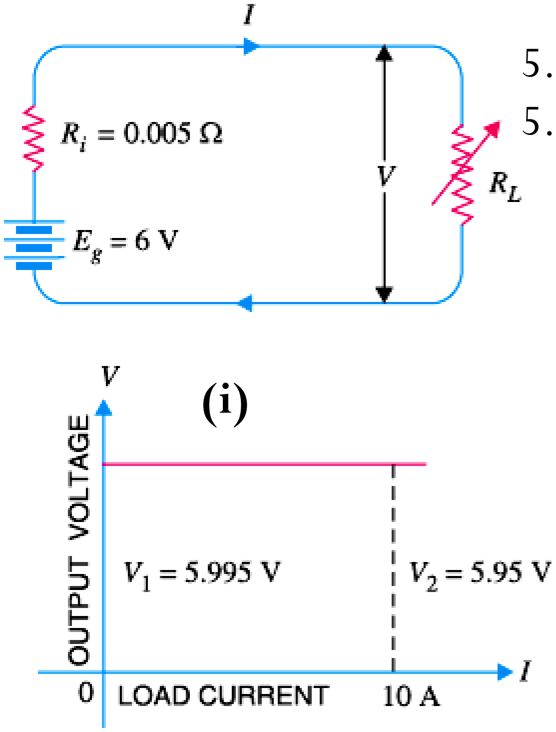
A voltage source which has very ***low internal impedance as compared with external load impedance*** is known as a constant voltage source.

In such a case, the output voltage nearly remains the same when load current changes. Fig. 3 (i) illustrates a constant voltage source. It is a d.c. source of 6 V with internal resistance $R_i = 0.005\ \Omega$. If the load current varies over a wide range of 1 to 10 A, for any of these values, the internal drop across R_i ($= 0.005\ \Omega$) is less than 0.05 volt. Therefore, the voltage output of the source is between 5.995 to 5.95 volts. This can be considered constant voltage compared with the wide variations in load current.

Fig. 3 (ii) shows the graph for a constant voltage source. It may be seen that the output voltage remains constant inspite of the changes in load current. Thus as the load current changes from 0 to 10 A, the output voltage essentially remains the same (i.e. $V_1 = V_2$).

A constant voltage source is represented as shown in Fig. 4.

Note: *It is clear from the above example that when* ***internal resistance of the source is quite small****, the* ***voltage drop in internal resistance is very low****. Therefore, the* ***terminal voltage substantially remains constant*** *and the source behaves as a constant voltage source irrespective of load current variations.*



5.995 Ω to 5.95 Ω

(ii)

Fig. 3

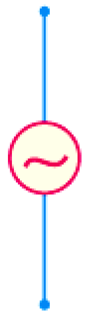


Fig. 4

Constant Current Source

A voltage source that has a very high internal impedance as compared with external load impedance is considered as a constant current source.

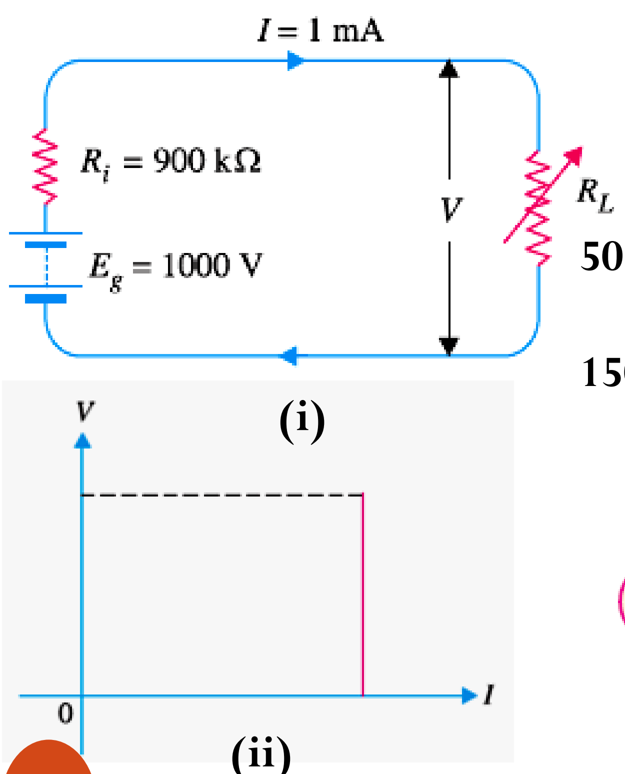
In such a case, the load current nearly remains the same when the output voltage changes. Fig. 5(i) illustrates a constant current source. It is a d.c. source of 1000 V with internal resistance R_i = 900 kΩ. Here, load R_L varies over 3 : 1 range from 50 kΩ to 150 kΩ. Over this variation of load R_L, the circuit current I is essentially constant at 1.05 to 0.95 mA or approximately 1 mA. It may be noted that output voltage V varies approximately in the same 3 : 1 range as R_L, although load current essentially remains constant at 1mA. The beautiful example of a constant current source is found in vacuum tube circuits where the tube acts as a generator having internal resistance as high as 1 MΩ.

Fig. 6(ii) shows the graph of a constant current source. It is clear that **current remains constant even when the output voltage changes** substantially.

Resistance in case of a d.c. source

Now $I = \dfrac{E_g}{R_L + R_i}$. Since $R_i >> R_L$, $I = \dfrac{E_g}{R_i}$

As both E_g and R_i are constants, I is constant.



50 kΩ to 150 kΩ

(i)

(ii)

Fig. 5

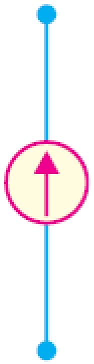


Fig.6 shows the symbol of a constant current source.

The following points may be noted regarding the constant current source :

(i) Due to ***high internal resistance of the source***, the ***load current remains essentially constant*** as the ***load R_L is varied***.

(ii) The ***output voltage varies*** approximately in the same range as R_L, although ***current remains constant.***

(iii) The ***output voltage V is much less*** than the ***generated voltage E_g*** because of ***high I R_i drop***.

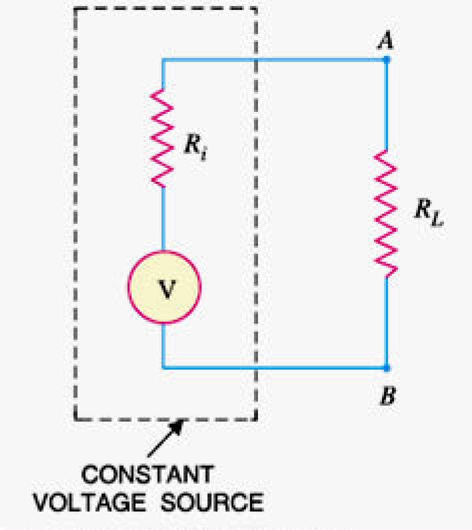
Conversion of Voltage Source into Current Source

Fig. 7 shows a constant voltage source with voltage V and internal resistance Ri. Fig. 8 shows its equivalent current source. It can be easily shown that the two circuits behave electrically the same way under all conditions.

(i) If in Fig. 7, the ***load is open-circuited*** (i.e. $R_L \rightarrow \infty$), then voltage across terminals A and B is V. If in Fig. 8, the load is ***open-circuited*** (i.e. $R_L \rightarrow \infty$), then all current I ($= V/R_i$) flows through R_i, yielding voltage across terminals AB = I R_i = V. Note that open-circuited voltage across AB is V for both the circuits and hence they are electrically equivalent.

(ii) If in Fig. 7, the ***load is short-circuited*** (i.e. $R_L = 0$), the short circuit current is given by: $\mathbf{I_{short} = V / R_i}$

If in Fig. 8, the ***load is short-circuited*** (i.e. $R_L = 0$), the current I ($= V/R_i$) bypasses ***R_i in favour of short-circuit***. It is clear that current ($= V/R_i$) is the same for the two circuits and hence they are electrically equivalent.



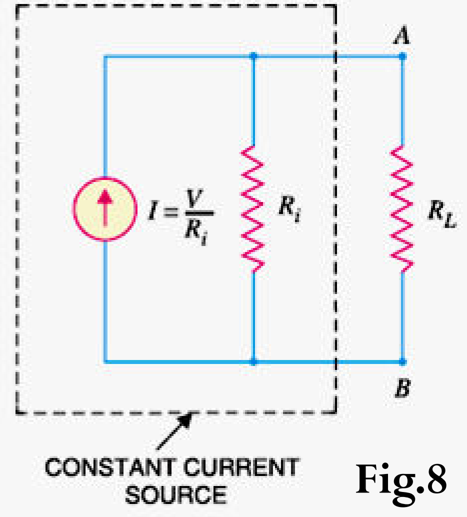


Fig.8

Thus to convert a constant voltage source into a constant current source, the following procedure may be adopted :

(a) Place a short-circuit across the two terminals in question (terminals AB in the present case) and find the short-circuit current. Let it be I. Then I is the current supplied by the equivalent current source.

(b) Measure the resistance at the terminals with load removed and sources of e.m.f.s replaced by their internal resistances if any. Let this resistance be R.

(c) Then equivalent current source can be represented by a single current source of magnitude I in parallel with resistance R.

Note. To convert a current source of magnitude I in parallel with resistance R into voltage source,

(a) Voltage of voltage source, V = I R, and

(b) Resistance of voltage source, R = R

Thus voltage source will be represented as voltage V in series with resistance R.

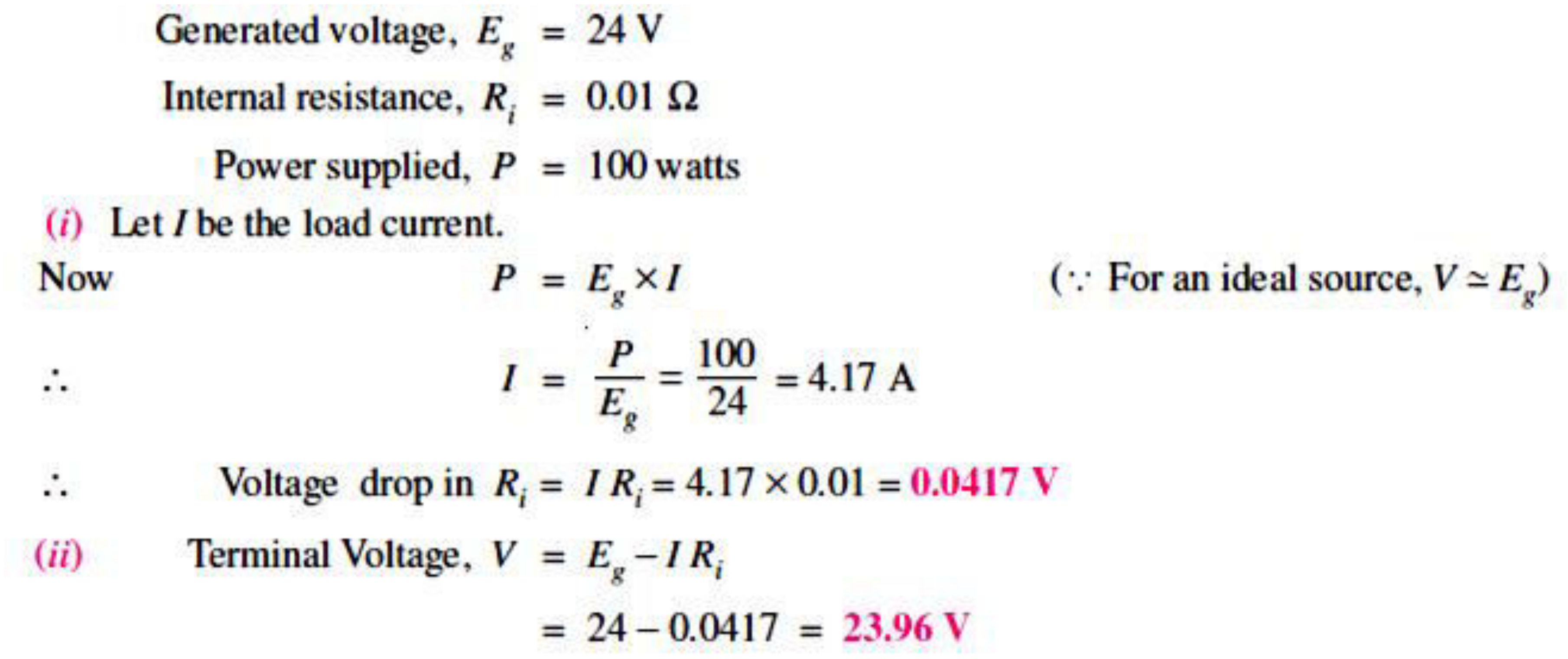
Problems

Example 1. A lead acid battery fitted in a truck develops 24V and has an internal resistance of 0.01Ω . It is used to supply current to head lights etc. If the total load is equal to 100 watts, find :

(i) voltage drop in internal resistance and

(ii) terminal voltage.

Solution



Generated voltage, $E_g = 24\text{ V}$

Internal resistance, $R_i = 0.01\ \Omega$

Power supplied, $P = 100\text{ watts}$

(*i*) Let I be the load current.

Now $P = E_g \times I$ $\quad$ ($\because$ For an ideal source, $V \simeq E_g$)

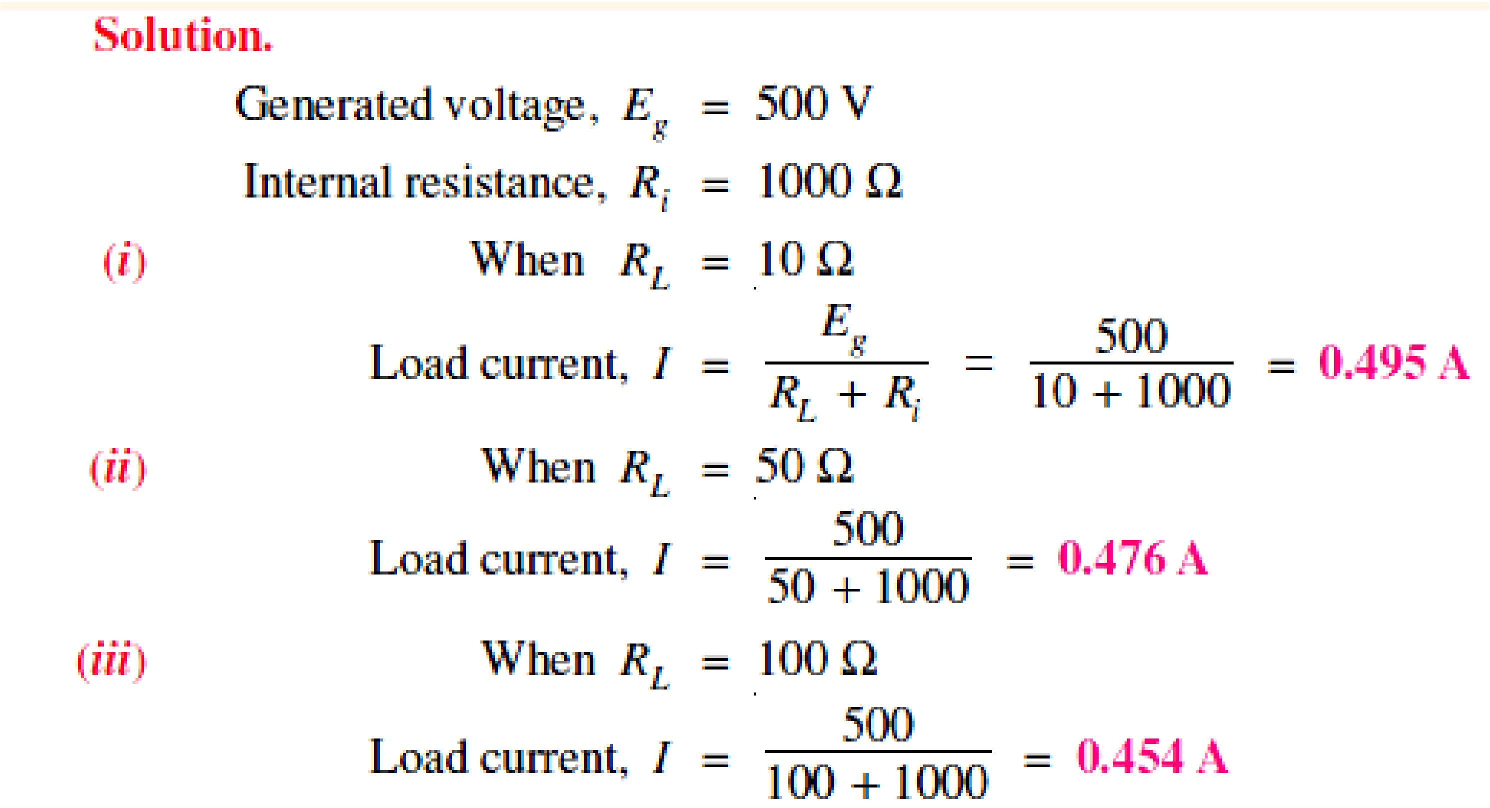
$$\therefore \quad I = \frac{P}{E_g} = \frac{100}{24} = 4.17\text{ A}$$

$$\therefore \quad \text{Voltage drop in } R_i = I\,R_i = 4.17 \times 0.01 = \mathbf{0.0417\ V}$$

(*ii*) Terminal Voltage, $V = E_g - I\,R_i$

$$= 24 - 0.0417 = \mathbf{23.96\ V}$$

Example 2. A d.c. source generating 500 V has an internal resistance of 1000 Ω. Find the load current if load resistance is (i) 10 Ω (ii) 50 Ω and (iii) 100 Ω.



Solution.

Generated voltage, $E_g = 500\text{ V}$

Internal resistance, $R_i = 1000\ \Omega$

(i) When $R_L = 10\ \Omega$

$$\text{Load current, } I = \frac{E_g}{R_L + R_i} = \frac{500}{10 + 1000} = \mathbf{0.495\ A}$$

(ii) When $R_L = 50\ \Omega$

$$\text{Load current, } I = \frac{500}{50 + 1000} = \mathbf{0.476\ A}$$

(iii) When $R_L = 100\ \Omega$

$$\text{Load current, } I = \frac{500}{100 + 1000} = \mathbf{0.454\ A}$$

It is clear from the above example that load current is essentially constant since $R_i >> R_L$.

Example 3. Convert the constant voltage source shown in Fig. (a) into constant current source.

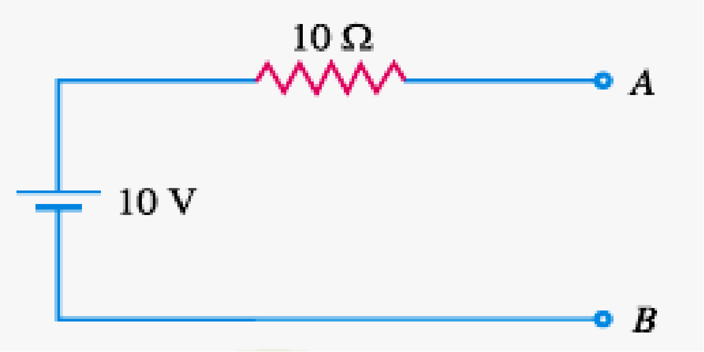


Fig. (a)

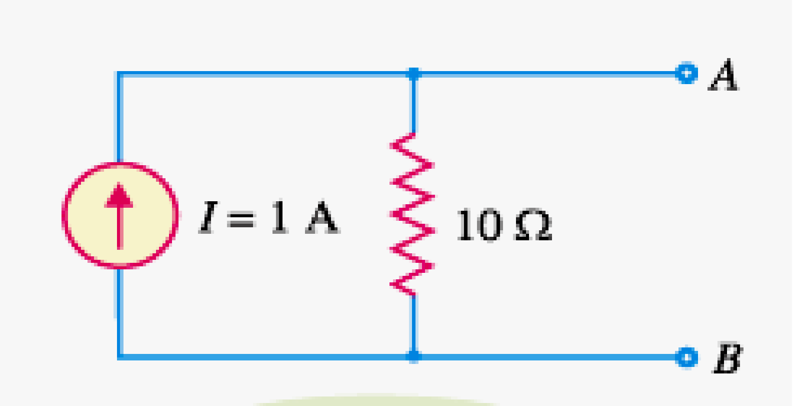


Fig. (b)

Solution.

The solution involves the following steps :

(i) Place a short across AB in Fig. (a) and find the short-circuit current I.

Clearly, $I = 10/10 = 1$ A

Therefore, the equivalent current source has a magnitude of 1 A.

(ii) Measure the resistance at terminals AB with load removed and 10 V source replaced by its internal resistance. The 10 V source has negligible resistance so that resistance at terminals AB is $R = 10\Omega$.

(iii) The equivalent current source is a source of 1 A in parallel with a resistance of 10Ω as shown in Fig. (b).