① (Subhash ch. Sen) **Electro magnetic induction** Deptt:- Physics(H)
SeM = II Paper = C3T

When the flux of electric induction ~~length~~ linked with a circuit chages & e.m.f in induced in the circuit carring a current to flow is called electro magnetic induction.

① **Faraday's law:**
It states that indued e.m.f is proporsotional to rate of change of flux.

or $e \propto \frac{d\psi}{dt}$

or, $e = K\frac{d\psi}{dt}$ [In S.I unit $k=1$]

or, $e = \frac{d\psi}{dt}$

② **Lenz's law**:- The direction of induced e.m.f is oppsite of change flux.

or, $e = -\frac{d\psi}{dt}$

⊛ Flux link with a circuit is $\psi = \int \vec{B}\cdot\vec{ds}$

Self inductance :-
When a circuit carries a current the magnetic flux produced by it is linked with the circuit it self. This pheonomina is called self inductance.

The flux linked with the circuit $\psi = \int \vec{B}\cdot\vec{ds}$

(i) $\psi \propto I$
$\therefore \psi = lI$ [l = Self inductance]

The flux linked with a circuit due to flow of unit current is called self inductance.

(ii) From Faraday's Lenz's law, $e = -\frac{d\psi}{dt} = -\frac{l\,dI}{dt}$

$\therefore$ Self inductance is the e.m.f induces in a circuit due to unit rate of change of current through it.

(iii) Induce e.m.f is $e = -\frac{l\,dI}{dt}$

Work done to move a further charge dq in the circuit is $dW = e\,dq = -l\frac{dI}{dt}\cdot dq = -l\,dI\frac{dq}{dt} = -lI\,dI$ $\left[\frac{dq}{dt} = I\right]$

$\therefore$ Total Work done in maintaining current I in the circuit is —

$$W = \int_0^I -lI\,dI = -l\frac{I^2}{2} = -\frac{1}{2}I^2 l$$

$\therefore$ Self inductance is two time the work done in maintaining unit current in the circuit.

$I = 1$

$$\boxed{l = 2W}$$

Unit of self inductance l:- $e = -l\frac{dI}{dt}$

$$\therefore l = \frac{-e}{\frac{dI}{dt}} = \frac{\text{volt}}{\text{Amp/sec}}$$

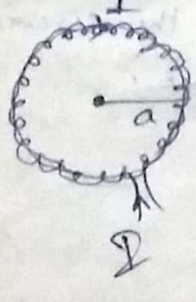
$= \text{volt}\cdot\text{sec}/\text{Amp}$

$= \text{Henry}$

Ex:- Self inductance of circuit

(i) **circular coil:-**

Let a circular coil radius 'a' and n turns per unit length carrying current I



The magnetic flux density or magnetic inductance at it centre is $B = \frac{\mu_0 I}{2a}$

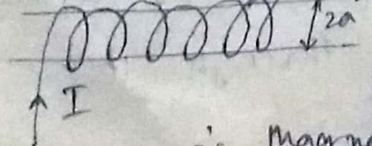
$\therefore$ Flux linked with the coil

$\psi = B\cdot A\cdot n$

$= \frac{\mu_0 I}{2a}\pi a^2\cdot n = lI$ [From def^n of self induc.]

$$\therefore l = \frac{\psi}{I} = \frac{\mu_0\pi a n}{2} = \frac{\mu_0}{2}\pi a n$$

(2) **Infinite Solenoid:-**



Let a Solenoid of radius a and n terms for unit length for a carrying current I

$\therefore$ magnetic flux density along the axis is

$B = \mu_0 I n$

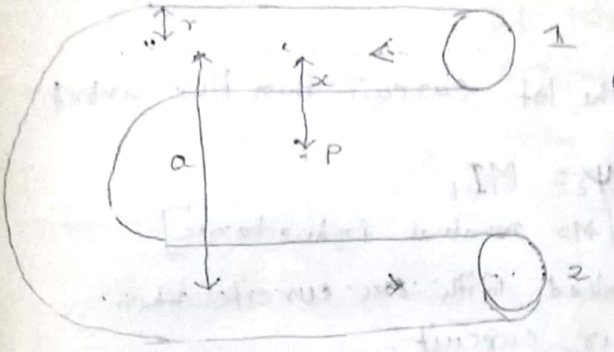
$\therefore$ Flux linked ~~with~~ of the solenoid is $\psi = B\cdot A\cdot n$

$\psi = B \cdot A \cdot n$

$= \mu_0 I \cdot n \times \pi a^2 \cdot n = l I$ [By def^n of Self inductance]

$\therefore l = \frac{\psi}{I} = \mu_0 n^2 \pi a^2$

Two identical parallel wire carring current in oppsite direction



We consider two llel wires each of radius are and seperated by carring current I in oppsite direction. P is a Point at a distance x form the first wire.

Magnetic induction at P due to current on first wire is $B_1 = \frac{\mu_0 I}{2\pi x}$, and for the current in 2nd wire

$B_2 = \frac{\mu_0 I}{2\pi(a-x)}$,

These are in same direction.

$\therefore$ Total magnetic induction $B = B_1 + B_2 = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{a-x}\right)$

We Consider an element of area P of unit length and with dx

$\therefore$ Magnetic flux linked with it $= B \times dx = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{a-x}\right) dx$

$\therefore$ Total flux linked per unit length of space bet^n the wire is

$$\psi = \frac{\mu_0 I}{2\pi}\int_r^{(a-r)}\left(\frac{1}{x} + \frac{1}{a-x}\right) dx$$

$$= \frac{\mu_0 I}{2\pi}\Big[\log x + \log(a-x)\Big]_r^{(a-r)} = \cancel{\frac{\mu_0 I}{2\pi}\left[\log\frac{a-r}{r} - \log\frac{r}{a-r}\right]}$$

$$= \cancel{\frac{\mu_0 I}{2\pi}\log\frac{a-r}{r}\frac{a-r}{r}} \quad = \frac{\mu_0 I}{2\pi}\left[\log\frac{x}{a-x}\right]_r^{a-r}$$

$$= \frac{\mu_0 I}{2\pi}\left[\log\frac{a-r}{a-a+r} - \log\frac{r}{a-r}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\log\frac{a-r}{r} + \log\frac{a-r}{r}\right]$$

$$= \frac{\mu_0 I}{\pi}\log_e\frac{a-r}{r} = lI$$

When the two wires are in contract $a = 2r$ self inducetence Per unit length, $l = \frac{\mu_0}{\pi} \log \frac{a-r}{r}$ $[\because a = 2r]$

$$= \frac{\mu_0}{\pi} \log 1 = 0$$

Mutual Inductance :-

When two circuit Placed togather closed by the flux Produce by current in one is linked to the other. This is called Mutual Induction.

If current I_1 in the 1st curcuit then flux linked with the 2nd curcuit is $\psi_2 = MI_1$

[M = muatual inductance]

(i) Mutual Inductance is the flux linked with one curcite when unit current flows through the other curcuit.

(ii) e_2 = induced e.m.f in the 2nd curcuit due to current I in 1st

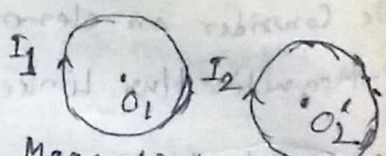
$$= M \frac{dI_1}{dt}$$

$$\therefore e_2 = M, \quad \left[\because \frac{dI_1}{dt} = 1\right]$$

(iii) Work done $W = \frac{1}{2} m I_1 I_2$

Proved that Mutual Inducetence betn two circuit are interechageable :-

let I_1 & I_2 be the current in the first and 2nd circuit respectirly and H_1 is Magnetic field at O_2 due to current I_1 in the first curcuit.



The Potential enery of the 2nd circuit with current I_2, in the field of first is

$$W_2 = -\vec{M_2} \cdot \vec{H_1}$$

$$= -\mu_0 I_2 \int d\vec{s_2} \cdot \vec{H_1} = -I_2 \int \mu_0 H_1 \cdot ds_2 = -I_2 \int \vec{B_1} \cdot d\vec{S_2}$$

$\psi_2 = M_{21} I_1$

$$= -I_2 \psi_2 \quad \left[\psi_2 = \vec{B_1} \cdot d\vec{s_2}\right]$$

$$= -M_{21} I_1 I_2$$

B = Magnetic field

ds = Area

Similarly Potential enery of first curcite in the field of 2nd is $W_1 = -M_{12} I_1 I_2$

Since the Potential energy is muatual

$$W_1 = W_2$$

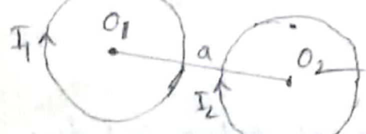
$\Rightarrow \; -M_{12} I_1 I_2 = -M_{21} I_1 I_2$

$\Rightarrow M_{12} = M_{21} = M$

$\therefore$ Mutual inductance of the 1st circuit due to 2nd is equal to Mutual inductance of the 2nd due to 1st

Ex:-1

Two coxial circular coil



Let two circular coil of radius r seperated by a distance a. current I_1 in the 1st circuit with n_1 no of turns and I_2 is the current in the 2nd circuit with n_2 no of turns.

Magnetic inductance, or, magnetic flux density at the centre of 2nd coil O_2 due to current I_1 in the 1st is

$$B_1 = \frac{\mu_0 I_1 r^2}{2} \cdot \frac{n_1}{(a^2+r^2)^{3/2}}$$

$\therefore$ Magnetic flux linked with 2nd coil is $\psi_2 = n_2 B_1 \pi r^2$

$$\psi_2 = \frac{n_1 n_2 \mu_0 I_1 r^2 \pi r^2}{2(a^2+r^2)^{3/2}} = M I_1$$

$$M = \frac{n_1 n_2 \mu_0 \pi r^4}{2(a^2+r^2)^{3/2}}$$

If the circuit are closed together.

$\therefore a \approx 0 \therefore M = \dfrac{n_1 n_2 \mu_0 \pi r^4}{2(r)^3} = \frac{1}{2} n_1 n_2 \pi \mu_0 r$

The self inductence at centere due to current I_1 is

$$l_1 = \frac{\mu_0 \pi}{2} n_1 r$$

Similarly $l_2 = \dfrac{\mu_0 \pi n_2 r}{2}$

$$\therefore l_1 \times l_2 = \frac{\mu_0^2 \pi^2 n_1 n_2 r^2}{4} = \frac{M^2}{n_1 n_2}$$

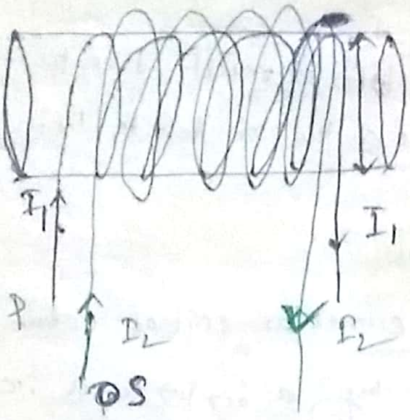
Now $\boxed{M = k\sqrt{l_1 l_2}}$ in general

k = cofficent of coupling

$\boxed{k<1}$

⑥

Solenoied



let the Primary coil of Solenoid have radius a, current I_1 and n_1 terms / unit length
Secondary coil S having radius a, current I_2 and n_2 terms / unit length.

Magnetic flux density along the axis of Soleneied due to current I_1 in P is $B_1 = \mu_0 I_1 n_1$

$\therefore$ Flux linked with Secondary is ψ_2

$$\psi_2 = B_1 \pi a^2 \cdot n_2$$

$$= \mu_0 I_1 n_1 n_2 \pi a^2 = M I_1$$

$$\therefore M = \mu_0 n_1 n_2 \pi a^2$$

combination of induction

(i) **In Series** :- let two circuit of inductance L_1 & L_2 are in series and closed togather if a current I through each of them in the same direction, the lines of force are in the same direction in both circuit

$\therefore$ Flux linked with the 1st circuit is $\psi_1 = l_1 I_1 + MI$

and flux linked with the 2nd circuit is $\psi_2 = l_2 I + MI$

Total flux linked with the System then $\psi = \psi_1 + \psi_2$

$$= \{(l_1 + l_2) + 2M\} I$$

If L_1 is the equivalent inductance of the system producing flux ψ for the current I then

$$\psi = L_1 I = \{(l_2 + l_2) + 2M\} I$$

$$\therefore L_1 = (l_1 + l_2) + 2M$$

② If the two circuit carrying current in opposite direction :-

The flux linked with the 1st circuit

$\psi_1 = l_1 I - MI$,

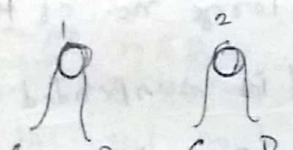
The flux linked with the 2nd circuit $\psi_2 = l_2 I - MI$

$\therefore \psi = \psi_1 + \psi_2 = \{(l_1+l_2) + 2M\} I$

$\therefore$ Equivalent inductence $\boxed{L_2 = (l_1+l_2-2M)}$

Problem

Two inductence of 1200 μH and 800 μH, Terminal [A, B] & [C, D] respectively when B & C are connacted the inductence betn A and D is 2.5 mH. What will be the inductence betn A & C If B is connacted with D



$l_1 = 1200\ \mu H$ = 1200 micro Henry

l_2 = 800 micro Henry

2.5 milli Henry = 2.5×1000 micro Henry

= 2500 μH.

1) $\therefore$ If B and C are connected then

$L_1 = l_1 + l_2 + 2M$.

$2500 = 1200 + 800 + 2M$ $\therefore M = 250\ \mu H$ = Mutual Inductance

2) $\therefore$ If A and C are connected then

$L_2 = l_1 + l_2 - 2M$

$= 1200 + 800 - 2 \times 250$

$= 1500\ \mu H$.