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- PAPER : CC9T
- TOPIC: GENERAL PROPERTIES OF ATOMIC NUCLEUS

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Nuclear size:

Most nuclei are approximately spherical. The average radius of a nucleus with A nucleons is $R = R_0A^{1/3}$, where $R_0 = 1.2*10^{-15}$ m. The volume of the nucleus is directly proportional to the total number of nucleons. This suggests that all nuclei have nearly the same density. Nucleons combine to form a nucleus as though they were tightly packed spheres.

Problem:

Find the radius of a ^{238}Pu nucleus. ^{238}Pu is a manufactured nuclide that is used as a power source on some space probes. I contains 238 nucleons.

Solution:
$R = R_0A^{1/3} = (1.2*10^{-15}\text{ m})*(238)^{1/3} = 7.4*10^{-15}$ m.

Problem:

Find the diameter of a ^{56}Fe nucleus.

Solution:
$R = R_0A^{1/3} = (1.2*10^{-15}\text{ m})*(56)^{1/3} = 4.6*10^{-15}$ m.
diameter = $2R = 9.2*10^{-15}$ m.

Nuclear Forces:

Nuclei are bound together by the residual strong force (nuclear force). The residual strong force is a minor residuum of the strong interaction which binds quarks together to form protons and neutrons. This force is much weaker *between* neutrons and protons because it is mostly neutralized within them, in the same way that electromagnetic forces *between* neutral atoms (such as van der Waals forces that act between two inert gas atoms) are much weaker than the electromagnetic forces that hold the parts of the atoms together internally (for example, the forces that hold the electrons in an inert gas atom bound to its nucleus).

The nuclear force is highly attractive at the distance of typical nucleon separation, and this overwhelms the repulsion between protons due to the electromagnetic force, thus allowing nuclei to exist. However, the residual strong force has a limited range because it decays quickly with distance (see Yukawa potential); thus only nuclei smaller than a certain size can be completely stable. The largest known completely stable nucleus (i.e. stable to alpha, beta, and gamma decay) is lead-208 which contains a total of 208 nucleons (126 neutrons and 82 protons). Nuclei larger than this maximum are unstable and tend to be increasingly short-lived with larger numbers of nucleons. However, bismuth-209 is also stable to beta decay and has the longest half-life to alpha decay of any known isotope, estimated at a billion times longer than the age of the universe.

The residual strong force is effective over a very short range (usually only a few femtometres (fm); roughly one or two nucleon diameters) and causes an attraction between any pair of nucleons. For example, between protons and neutrons to form [NP] deuteron, and also between protons and protons, and neutrons and neutrons.

Halo nuclei and nuclear force range limits

The effective absolute limit of the range of the nuclear force (also known as residual strong force) is represented by halo nuclei such as lithium-11 or boron-14, in which dineutrons, or other collections of neutrons, orbit at distances of about 10 fm (roughly similar to the 8 fm radius of the nucleus of uranium-238). These nuclei are not maximally dense. Halo nuclei form at the extreme edges of the chart of the nuclides—the neutron drip line and proton drip line—and are all unstable with short half-lives, measured in milliseconds; for example, lithium-11 has a half-life of 8.8 ms.

Halos in effect represent an excited state with nucleons in an outer quantum shell which has unfilled energy levels "below" it (both in terms of radius and energy). The halo may be made of either neutrons [NN, NNN] or protons [PP, PPP]. Nuclei which have a single neutron halo include ^{11}Be and ^{19}C. A two-neutron halo is exhibited by ^{6}He, ^{11}Li, ^{17}B, ^{19}B and ^{22}C. Two-neutron halo nuclei break into three fragments, never two, and are called *Borromean nuclei* because of this behavior (referring to a system of three interlocked rings in which breaking any ring frees both of the others). ^{8}He and ^{14}Be both exhibit a four-neutron halo. Nuclei which have a proton halo include ^{8}B and ^{26}P. A two-proton halo is exhibited by ^{17}Ne and ^{27}S. Proton halos are expected to be more rare and unstable than the neutron examples, because of the repulsive electromagnetic forces of the excess proton(s).

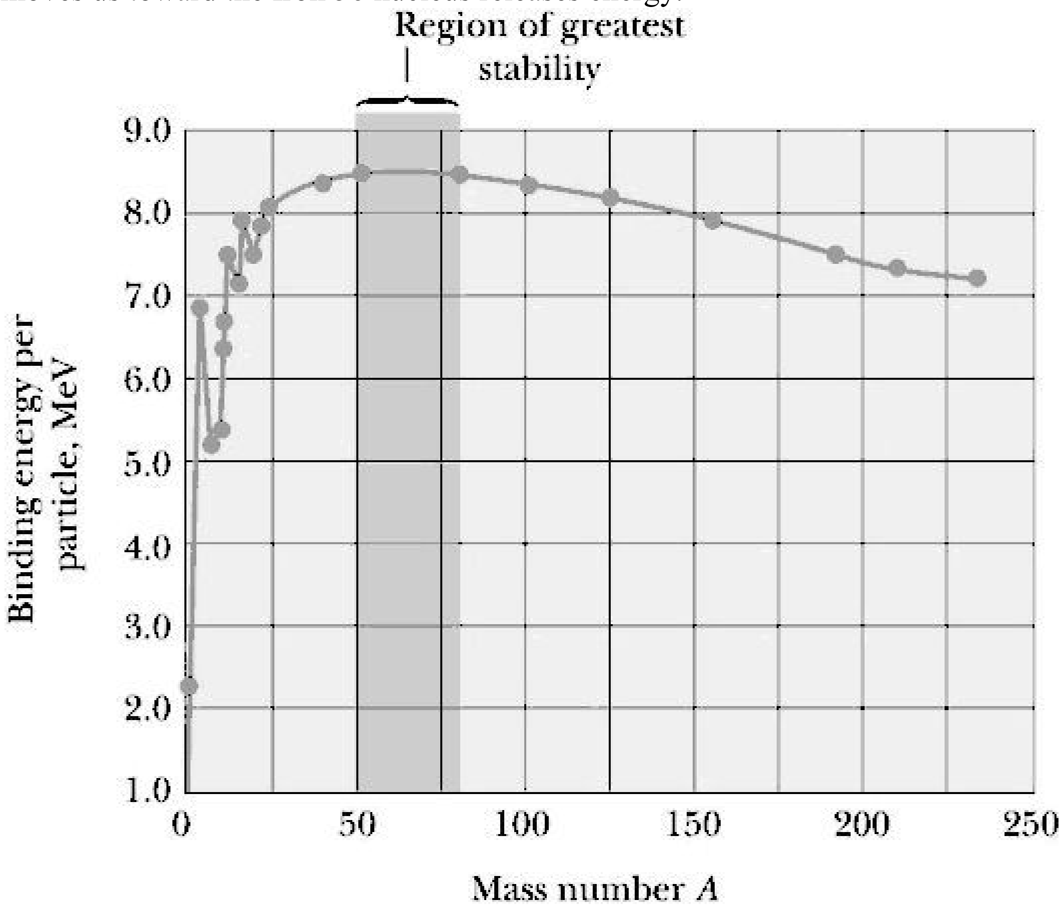
Binding energy:

The best way to see the competition between the attractive nuclear force and the electric repulsive force inside atomic nuclei is to look at **nuclear binding energies**. The binding energy per nucleon (proton or neutron) represents how much energy we would have to supply to pull the nucleus apart into separate free nucleons. The nuclear force tries to hold the nucleus together and therefore increases the binding energy. The electrostatic force, which pushes the protons apart, decreases the binding energy. We calculate the binding energy of a nucleus by subtracting the rest energy of the nucleus from the sum of the rest energies of the protons and neutrons that make up the nucleus. We then divide by the number of nucleons to get the **binding energy per nucleon**. For the deuteron the binding energy per nucleon is therefore 1.1 MeV.

The figure on the right is a plot of the binding energy, per nucleon, of the most stable nuclei for each element. The peak of that curve is at the Iron 56 nucleus, no other nucleus is more tightly bound. Except for light nuclei, the binding energy is about 8 MeV per nucleon.

Moving towards higher binding energy represents a release of energy. There are two ways to do this. We can start with light nuclei and put them together to form heavier nuclei, moving in and up from the left side in the figure. This process is called nuclear fusion. Or we can split apart heavy nuclei moving in and up from the right side. This process is called nuclear fission. Fusion represents the release of nuclear potential energy, while fission represents the release of electric potential energy. When we get to Iron 56, there is no energy to be released either by fusion or fission.

The importance of knowing the nuclear binding energy per nucleon is that it tells us whether energy will be released in a particular nuclear reaction. If the somewhat weakly bound uranium nucleus (7.41 MeV/ nucleon) splits into two more tightly bound nuclei like cesium (8.16 MeV/nucleon) and zirconium (8.41 MeV/ nucleon), energy is released. At the other end of the graph, if we combine two weakly bound deuterium nuclei (2.8 MeV/nucleon) to form a more tightly bound Helium 4 nucleus (7.1 MeV/nucleon), energy is also released. Any reaction that moves us toward the Iron 56 nucleus releases energy.



Problem:

Given the mass of the alpha particle, $mc^2 = 3727.38$ MeV, find the binding energy per nucleon.

Solution:
The sum of the masses of two protons and two neutrons is 3755.66 MeV.
The binding energy of Helium 4 is (3755.66 - 3727.38) MeV = 28.28 MeV.
The binding energy per nucleon is 28.28 eV/4 = 7.07 MeV.

Binding energy formula:

Atomic and nuclear data tables often list the mass of the neutral atom (not that of the nucleus) in atomic mass units (u). Atomic masses include the masses of the atomic electrons, and thus are not equal to the nuclear masses. One u is (1/12)th of the mass of the neutral carbon **atom** , 1 u =

$(1/12)m_{12C}$. This can easily be converted to SI units. One mole of ^{12}C has a mass of 0.012 kg, and contains Avogadro's number particles, thus

$$1\ u = (0.001\ kg)/N_A = 1.66054*10^{-27}\ kg = 931.494\ MeV/c^2.$$

We can write down a formula for the binding energy of a nucleus in terms of the nuclear masses or in terms of the atomic masses. The binding energy is defined as the the total mass energy of constituent nucleons minus the mass energy of the nucleus. It is the total energy one needs to invest to decompose the nucleus into nucleons.

In terms of the nuclear masses, we write for the binding energy B(Z,N) of a nucleus with Z protons and N neutrons

$$B(Z,N) = c^2(Z*m_p + N*m_n - M_{nuc}(Z,N)).$$

In terms of the atomic masses, we write

$$B(Z,N) = c^2(Z*m_H + N*m_n - M_{atom}(Z,N)).$$

The masses of the Z electrons cancel out and the difference in binding energies of the electrons in the different atoms (~eV) is negligible compared to the nuclear binding energy (~MeV).

Problem:

What is the binding energy per nucleon for ^{120}Sn?

Solution:
Using an atomic and nuclear data table we find for ^{120}Sn:
M_{atom} = 119.902199 u, Z = 50, N = 70, m_H = 1.007825 u, m_n = 1.008665 u.
$B(Z,N)/c^2 = (Z*m_H + N*m_n - M_{atom}(Z,N)) = (50*1.007825 + 70*1.008665 - 119.902199)\ u = 1.0956\ u$.
$B(Z,N) = (1.0956\ u)c^2 * (931.494\ MeV/c^2)/u = 1020.5\ MeV$.
Binding energy per nucleon = 1020.5 MeV/120 = 8.5 MeV

Problem:

What is the binding energy per nucleon for ^{262}Bh? The mass of the atom is 262.1231 u.

Solution:
Using an atomic and nuclear data table we find for ^{262}Bh (Bohrium):
Z = 107, N = 155.
$B(Z,N)/c^2 = (Z*m_H + N*m_n - M_{atom}(Z,N)) = (107*1.007825 + 155*1.008665 - 262.1231)\ u = 2.05725\ u$.
$B(Z,N) = (2.05725\ u)c^2 * (931.494\ MeV/c^2)/u = 1916.316\ MeV$.
Binding energy per nucleon = 1916.316 MeV/262 = 7.3 MeV.

Semi-empirical mass formula

In nuclear physics, the **semi-empirical mass formula** (**SEMF**) (sometimes also called the **Weizsäcker formula**, **Bethe–Weizsäcker formula**, or **Bethe–Weizsäcker mass formula** to distinguish it from the Bethe–Weizsäcker process) is used to approximate the mass and various other properties of an atomic nucleus from its number of protons and neutrons. As the name suggests, it is based partly on theory and partly on empirical measurements. The formula represents the **liquid drop model** proposed by George Gamow , which can account for most of the terms in the formula and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by German physicist Carl Friedrich von Weizsäcker and although refinements have been made to the coefficients over the years, the structure of the formula remains the same today.

The formula gives a good approximation for atomic masses and thereby other effects. However, it fails to explain the existence of lines of greater binding energy at certain numbers of protons and neutrons. These numbers, known as magic numbers, are the foundation of the nuclear shell model.

The formula

The mass of an atomic nucleus, for N neutrons, Z protons, and therefore $A = N + Z$ nucleons, is given by

$$m = Zm_p + Nm_n - \frac{E_B(N,Z)}{c^2}$$

where m_p and m_n are the rest mass of a proton and a neutron, respectively, and E_B is the binding energy of the nucleus. The semi-empirical mass formula states the binding energy is:

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} - \delta(N,Z)$$ [4]

The $\delta(N,Z)$ term, depending on N and $\delta_0 = a_P A^{k_P}$., is either zero or $\pm\delta_0$, where $\delta_0 = a_P A^{k_P}$ for some exponent k_P

Volume term

The term $a_V A$ is known as the *volume term*. The volume of the nucleus is proportional to A, so this term is proportional to the volume, hence the name.

The basis for this term is the strong nuclear force. The strong force affects both protons and neutrons, and as expected, this term is independent of Z. Because the number of pairs that can be taken from A particles is $\frac{A(A-1)}{2}$, one might expect a term proportional to A^2. However, the strong force has a very limited range, and a given nucleon may only interact strongly with its nearest neighbors and next nearest neighbors. Therefore, the number of pairs of particles that actually interact is roughly proportional to A, giving the volume term its form.

The coefficient a_V is smaller than the binding energy possessed by the nucleons with respect to their neighbors (E_b), which is of order of 40 MeV. This is because the larger the number of nucleons in the nucleus, the larger their kinetic energy is, due to the Pauli exclusion principle. If one treats the nucleus as a Fermi ball of A nucleons, with equal numbers of protons and neutrons, then the total kinetic energy is $\frac{3}{5}A\varepsilon_F$, with ε_F the Fermi energy which is estimated as 28 MeV. Thus the expected value of a_V in this model is $E_b - \frac{3}{5}\varepsilon_F \sim 17\ \mathrm{MeV}$, not far from the measured value.

Surface term

The term $a_S A^{2/3}$ is known as the *surface term*. This term, also based on the strong force, is a correction to the volume term.

The volume term suggests that each nucleon interacts with a constant number of nucleons, independent of A. While this is very nearly true for nucleons deep within the nucleus, those nucleons on the surface of the nucleus have fewer nearest neighbors, justifying this correction. This can also be thought of as a surface tension term, and indeed a similar mechanism creates surface tension in liquids.

If the volume of the nucleus is proportional to A, then the radius should be proportional to $A^{1/3}$ and the surface area to $A^{2/3}$. This explains why the surface term is proportional to $A^{2/3}$. It can also be deduced that a_S should have a similar order of magnitude as a_V.

Coulomb term

The term $a_C \frac{Z(Z-1)}{A^{1/3}}$ or $a_C \frac{Z^2}{A^{1/3}}$ is known as the *Coulomb* or *electrostatic term*.

The basis for this term is the electrostatic repulsion between protons. To a very rough approximation, the nucleus can be considered a sphere of uniform charge density. The potential energy of such a charge distribution can be shown to be

$$E = \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q^2}{R}$$

where Q is the total charge and R is the radius of the sphere. Identifying Q with Ze, and noting as above that the radius is proportional to $A^{1/3}$, we get close to the form of the Coulomb term. However, because electrostatic repulsion will only exist for more than one proton, Z^2 becomes $Z(Z-1)$. The value of a_C can be approximately calculated using the equation above:

Asymmetry term

The term $a_A \frac{(A-2Z)^2}{A}$ is known as the *asymmetry term* (or *Pauli term*). Note that as $A = N + Z$, the parenthesized expression can be rewritten as $(N - Z)$. The form $(A - 2Z)$ is used to keep the dependence on A explicit, as it will be important for a number of uses of the formula.

Pairing term

The term $\delta(A, Z)$ is known as the *pairing term* (possibly also known as the pairwise interaction). This term captures the effect of spin-coupling. It is given by:[6]

$$\delta(A,Z) = \begin{cases} +\delta_0 & Z, N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ -\delta_0 & Z, N \text{ odd } (A \text{ even}) \end{cases}$$

where δ_0 is found empirically to have a value of about 1000 keV, slowly decreasing with mass number A. The dependence on mass number is commonly parametrized as

$$\delta_0 = a_P A^{k_P}.$$

The liquid drop model

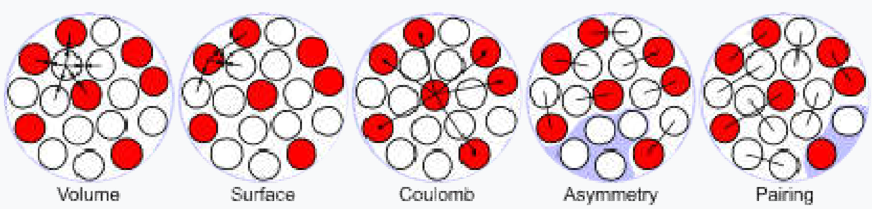


Illustration of the terms of the semi-empirical mass formula in the liquid drop model of the atomic nucleus.

The liquid drop model was first proposed by George Gamow and further developed by Niels Bohr and John Archibald Wheeler. It treats the nucleus as a drop of incompressible fluid of very high density, held together by the nuclear force (a residual effect of the strong force), there is a similarity to the structure of a spherical liquid drop. While a crude model, the liquid drop model accounts for the spherical shape of most nuclei, and makes a rough prediction of binding energy.

The corresponding mass formula is defined purely in terms of the numbers of protons and neutrons it contains. The original Weizsäcker formula defines five terms:

- *Volume energy*, when an assembly of nucleons of the same size is packed together into the smallest volume, each interior nucleon has a certain number of other nucleons in contact with it. So, this nuclear energy is proportional to the volume.
- *Surface energy* corrects for the previous assumption made that every nucleon interacts with the same number of other nucleons. This term is negative and proportional to the surface area, and is therefore roughly equivalent to liquid surface tension.
- *Coulomb energy*, the potential energy from each pair of protons. As this is a repulsive force, the binding energy is reduced.
- *Asymmetry energy* (also called Pauli Energy), which accounts for the Pauli exclusion principle. Unequal numbers of neutrons and protons imply filling higher energy levels for one type of particle, while leaving lower energy levels vacant for the other type.
- *Pairing energy*, which accounts for the tendency of proton pairs and neutron pairs to occur. An even number of particles is more stable than an odd number due to spin coupling.

Assignment:

1. Why are the most stable nuclei found in the region near A=60?
2. Chlorine 33 decay by positron emission with a maximum energy of 4.3MeV.calculate the radius of the nucleus from this .
3. Calculate the binding energy of the following isobars and their binding energy per nucleon. a. Ni 64 =63.927958u. b. Cu 64=63.929759u.(β- active)
4. Find the energy release ,if two H^2 nuclei fuse together to form He^4 nucleus. The binding energy per nucleon of H and He is 1.1 MeV and 7.0MeV res.