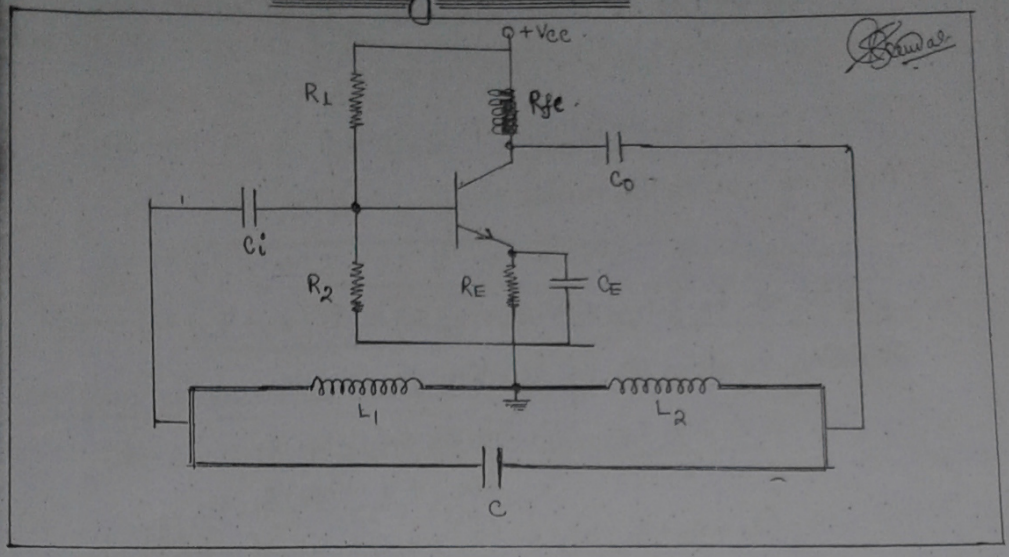
Hartley Oscillator <V.U-2002, 2004>



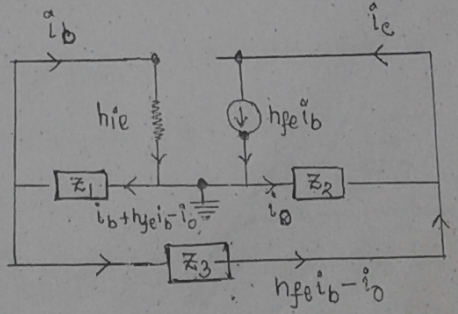
The circuit diagram of a Hartley oscillator using CE transistor and tuned (LC) network is shown in figure.

The resistors R_1, R_2, $R_E - C_E$ combination together with V_{BE} collector power supply V_{cc} provide stabilized self bias. C_i and C_o act as blocking or bypass capacitors. The radio frequency choke (Rfc) prevents RF current from reaching the collector power supply.; it also provides d.c. load to the collector. The capacitor C and the inductors L_1 and L_2 form the frequency determining tank circuit.

Now if the amplifier provides sufficient gain to produce unity loop gain i.e $\beta A = 1$, then sustained oscillation will be obtained.

Frequency of oscillation - calculation:

To analyse the performance of the circuit, we consider the approximate h parameter equivalent circuit of the oscillator.



Where $Z_1 = jX_1 = j\omega L_1$

$Z_2 = jX_2 = j\omega L_2$

$Z_3 = jX_3 = -\frac{j}{\omega C}$

Applying KVL to the input circuit, we get –

$$i_b \cdot h_{ie} + (i_b + h_{fe} i_b - i_o) Z_1 = 0$$

$$\Rightarrow i_b (h_{ie} + Z_1 + h_{fe} Z_1) = i_o \cdot Z_1 \quad \text{——} (1)$$

Applying KVL to the output circuit, we get –

$$i_o \cdot Z_2 - (h_{fe} i_b - i_o) Z_3 - (i_b + h_{fe} i_b - i_o) Z_1 = 0$$

$$\Rightarrow i_b (h_{fe} Z_3 + Z_1 + h_{fe} Z_1) = i_o (Z_1 + Z_2 + Z_3) \quad \text{——} (2)$$

Deviding equation (2) by equation (1), we get –

$$\frac{Z_1 + Z_2 + Z_3}{Z_1} = \frac{h_{fe} Z_3 + (1 + h_{fe}) Z_1}{h_{ie} + (1 + h_{fe}) Z_1}$$

$$\Rightarrow (Z_1 + Z_2 + Z_3) h_{ie} + (Z_1^2 + Z_1 Z_2 + Z_1 Z_3)(1 + h_{fe})$$

$$= h_{fe} Z_1 Z_3 + Z_1^2 (1 + h_{fe})$$

$$\Rightarrow (Z_1 + Z_2 + Z_3) h_{ie} + Z_1 Z_2 (1 + h_{fe}) + Z_1 Z_3 = 0 \quad \text{——} (3)$$

Now substituting $Z_1 = j X_1$

$Z_2 = j X_2$

$Z_3 = j X_3$

we get – $j (X_1 + X_2 + X_3) h_{ie} - [X_1 X_2 (1 + h_{fe}) + X_1 X_3] = 0 \quad \text{——} (4)$

Above equation holds good if real part and imaginary part separately equal to zero, i.e $(1 + h_{fe}) X_1 X_2 + X_1 X_3 = 0 \quad \text{——} (5)$

and $(X_1 + X_2 + X_3) h_{ie} = 0 \quad \text{——} (6)$

Since $h_{ie} \neq 0$, $X_1 + X_2 + X_3 = 0$

$$\Rightarrow \omega L_1 + \omega L_2 - \frac{1}{\omega C} = 0$$

$$\Rightarrow \omega^2 = \frac{1}{C(L_1 + L_2)}$$

$\therefore$ Frequency of oscillation $\boxed{f = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}}$

* Better: $X_1 = \omega(L_1$
$X_2 = \omega(L_2$
$X_3 = -\frac{1}{\omega}$

Then $f = \frac{1}{2\pi\sqrt{C(L_1}}$

Note: If there is mutual inductive effect between the inductors then

$$\left.\begin{aligned} X_1 &= \omega(L_1 \pm M) \\ X_2 &= \omega(L_2 \pm M) \\ X_3 &= -\frac{1}{\omega C} \end{aligned}\right\} \quad \omega^2 =$$

then $\boxed{f = \frac{1}{2\pi\sqrt{C(L_1 + L_2 \pm 2M)}}} \Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{C \cdot L_S}}}$

Condition of sustained oscillation:

The condition of sustained oscillation is,

$$(1 + h_{fe}) X_1 X_2 + X_1 X_3 = 0$$

$$\Rightarrow (1 + h_{fe}) X_2 + X_3 = 0 \quad \text{as } X_1 \neq 0$$

$$\Rightarrow (1 + h_{fe}) \omega L_2 + \frac{-1}{\omega C} = 0$$

$$\Rightarrow \omega^2 = \frac{1}{C L_2 (1 + h_{fe})}$$

$$\Rightarrow \frac{1}{C(L_1 + L_2)} = \frac{1}{C L_2 (1 + h_{fe})}$$

$$\Rightarrow L_1 + L_2 = L_2 + h_{fe} L_2$$

$$\Rightarrow h_{fe} = \frac{L_1}{L_2}$$

Thus the condition of sustained oscillation is $\boxed{h_{fe} \geq \frac{L_1}{L_2}}$

Note: If there is mutual inductive effect between the inductors then

$$\left.\begin{aligned} X_1 &= \omega(L_1 \pm M) \\ X_2 &= \omega(L_2 \pm M) \\ X_3 &= -\frac{1}{\omega C} \end{aligned}\right\}$$

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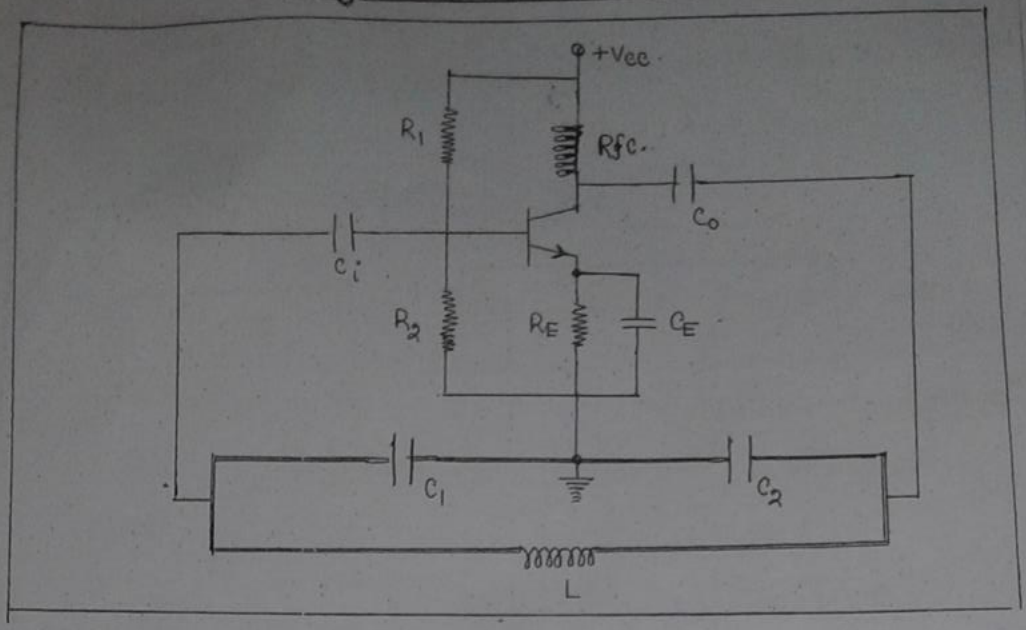
Colpitts Oscillator <V.U - 2006>



The circuit diagram of a Colpitts oscillator using CE transistor and tuned (LC) network is shown in figure.

The resistors R_1, R_2, $R_E - C_E$ combination together with collector power supply V_{cc} provide stabilized self bias. C_i and C_o act as blocking or bypass capacitors. The radio frequency choke prevents RF current from reaching the collector power supply; it also provides d.c load to the collector. The inductor L and the capacitors C_1 and C_2 form the frequency determining tank circuit.

Now if the amplifier provides sufficient gain to produce unity loop gain i.e $\beta A = 1$, then sustained oscillation will be obtained.

Frequency of oscillation calculation:

Same as Hartley oscillator: .....

$$X_1 + X_2 + X_3 = 0 \quad \cdots$$

$$\Rightarrow -\frac{1}{\omega C_1} + -\frac{1}{\omega C_2} + \omega L = 0 \cdot \Rightarrow \omega^2 = \frac{1}{L}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\begin{cases} X_1 = -\frac{1}{\omega C_1} \\ X_2 = -\frac{1}{\omega C_2} \\ X_3 = \omega L \end{cases}$$

$\therefore$ Frequency of oscillation, $\boxed{f = \frac{1}{2\pi\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}}$

$\therefore$ Frequency of oscillation, $\omega = \sqrt{\frac{C_1+C_2}{L \cdot C_1 C_2}}$.

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{C_1+C_2}{L \cdot C_1 C_2}}.$$

$$\Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{L \cdot \frac{C_1 C_2}{C_1+C_2}}}} \Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{LC_T}}}$$

● **Condition of sustained oscillation:**

The condition of sustained oscillation,

$$(1+h_{fe})X_2 + X_3 = 0$$

$$\Rightarrow -(1+h_{fe})\frac{1}{\omega C_2} + \omega L = 0$$

$$\Rightarrow \omega^2 = \frac{1+h_{fe}}{LC_2}$$

$$\Rightarrow \frac{1}{L}\left(\frac{1}{C_1}+\frac{1}{C_2}\right) = \frac{1+h_{fe}}{LC_2}$$

$$\Rightarrow \frac{1}{C_1} = \frac{h_{fe}}{C_2}$$

$$\Rightarrow h_{fe} = \frac{C_2}{C_1}$$

Thus, the condition of sustained oscillation, $\boxed{h_{fe} \geq \frac{C_2}{C_1}}$

● **What is the difference between Colpitts and Hartley oscillator?** V.U-2006

i) In Hartley oscillator the frequency determining tank circuit consists of two inductors and a capacitor.

Whereas in Colpitts oscillator the frequency determining tank circuit, the inductor and capacitor positions are interchanged to that of Hartley oscillator.

ii) In Hartley oscillator, frequency of oscillation, $f_L = \frac{1}{2\pi\sqrt{C(L_1+L_2)}}$

In Colpitts oscillator, frequency of oscillation, $f_C = \frac{1}{2\pi\sqrt{L \cdot \frac{C_1 C_2}{C_1+C_2}}}$

iii) Condition of sustained oscillation,

- H – oscillator, $h_{fe} \geq \frac{L_1}{L_2}$
- C – oscillator, $h_{fe} \geq C_2/C_1$.

Problem: (V.U~2000) In Hartley oscillator $L_1 = 200\ mH$, $L_2 = 100\ mH$ and $C = 40\ \mu F$. At what frequency the oscillation occour?

Solution: The frequency of oscillation in Hartley oscillator is given by, $f = \frac{1}{2\pi\sqrt{(L_1+L_2)C}}$

$$= \frac{1}{2\times 3.14\sqrt{(200\times10^{-3}+100\times10^{-3})\times40\times10^{-6}}}$$

$$= \frac{1}{6.28\times\sqrt{3\times10^{-1}\times40\times10^{-6}}}$$

$$= \frac{10^3}{6.28\times2\sqrt{3}}$$

$$= \frac{1000}{6.28\times2\times1.732}$$

$$= 45.97\ Hz$$

Problem: A coil of 10 μF used in Colpitts oscillator generates a wave of frequency 16 MHz. The transistor has $h_{fe}=50$ and $\Delta h_e = 0.5$. Find the value of capacitance.

Solution: For a Colpitts oscillator, $\omega^2 = \frac{1}{L}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$

$$\Rightarrow (2\pi\times16\times10^6)^2 = \frac{1}{10\times10^{-6}}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$$

$$\Rightarrow \frac{1}{C_1}+\frac{1}{C_2} = 4\pi^2\times256\times10^{7}.$$

Condition of sustained oscillation $\frac{C_1}{C_2} \simeq h_{fe}$

Actually, $\frac{C_1}{C_2} = \frac{h_{fe}}{\Delta h_e}$ $\quad(\Delta h_e = h_{ie}h_{oe} - h_{fe}h_{re})$

$$\Rightarrow \frac{C_1}{C_2} = \frac{50}{0.5} \quad \therefore C_1 = 100\,C_2.$$

$$\therefore \frac{1}{100\,C_2}+\frac{1}{C_2} = 4\pi^2\times256\times10^7 \Rightarrow C_2 = \frac{101}{100\times1024\times\pi^2\times10^7}.$$

$$= 10\times10^{-6}\,F = 10\ pF.$$

$$\therefore C_1 = 100\times10\ pF = 1000\ pF$$