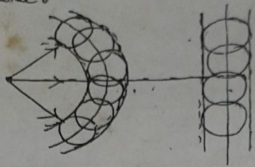
Wave Theory — OPTICS

Q. What is Huygen's principle for the propagation of light?

Ans. Huygen's principle :—

If S is the source of light, it sends energy in the form of waves in all directions. After an interval of time t all the particles of the medium lying on the surface AB are vibrating in the same phase. AB is thus the portion which has been drawn with S as centre and radius SA equal to ct where c is the velocity of propagation of the wave. The surface AB is called the primary wave front.

A wavefront is defined as the locus of all points having in the same phase.



Above figure shows that, the rays of light, converging to or diverging from a point, gives rise to a spherical wavefront and a parallel beam of light gives rise to a plane wave front.

The direction in which the disturbance is propagated in a homogeneous medium is called the ray. It is always normal to the wavefront.

According to Huygen's principle —

All points on the primary wavefront are considered to be centres of disturbances and send out secondary waves in all directions which travel through space with the same velocity in an isotropic medium.

Interference

2

Interference of light —

When two wave trains of same wavelength and preferably of same amplitude, propagating in a medium along the same direction superpose at a point, we get maximum or minimum intensity of light. This phenomena of production of maximum or minimum intensity of light according as they meet in same phase or in opposite phase, is called interference of light.

Coherent source :— When the monochromatic waves from two sources have constant phase difference for all times, then the sources are called coherent sources.

Remember that — Intensity $\propto$ (Amplitude)^2

i.e. $I \propto A^2$

i.e. $I = KA^2$.

Define constructive and destructive interference of light.

(i) **Constructive interference** — When two light waves from two coherent sources meet at a point in the same phase, then we get maximum intensity of light which is known as constructive interference of light.

(ii) **Destructive interference** — When two light waves from two coherent sources meet at a point in the opposite phase, then we get minimum intensity of light, which is known as destructive interference of light

Remember that ——

Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

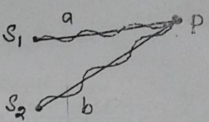
or path difference $= \frac{\lambda}{2\pi} \times$ phase difference

(B) Theory of interference of light

< Condition of constructive and destructive interference >

Let us consider two coherent sources S_1 and S_2 which produces monochromatic waves of light having wavelength λ and amplitudes a and b respectively. They proceed in the same direction and superpose at a point P.

Let y_1 and y_2 be the displacements of the particles at P in time t,



then
$$y_1 = a\sin\omega t$$
$$y_2 = b\sin(\omega t + \phi)$$

where ϕ is the phase difference between two wave

On superposition, the resultant displacement,

$$y = y_1 + y_2$$
$$= a\sin\omega t + b\sin(\omega t + \phi)$$
$$= a\sin\omega t + b\sin\omega t\cos\phi + b\cos\omega t . \sin\phi$$
$$= (a + b\cos\phi)\sin\omega t + b\sin\phi . \cos\omega t$$

Let
$$\left.\begin{aligned} a + b\cos\phi &= A\cos\delta \\ b\sin\phi &= A\sin\delta \end{aligned}\right\}$$

$$= A\sin\omega t\cos\delta + A\cos\omega t . \sin\delta$$

$$\therefore y = A\sin(\omega t + \delta)$$

Amplitude of the resulting wave,

$$\boxed{A = (a^2 + b^2 + 2ab\cos\phi)^{1/2}}$$

and
$$\delta = \tan^{-1}\left(\frac{b\sin\phi}{a + b\cos\phi}\right).$$

(4) (i) Condition of constructive interference

For constructive interference, intensity and hence amplitude is maximum.

i.e $A = maximum$.

i.e $\cos\phi = 1 = \cos 2n\pi$ where $n = 0, \pm1, \pm2 \cdots$

i.e $\phi = 2n\pi$

i.e $\boxed{\text{Phase diff} = 2n \times \pi}$

or $\boxed{\text{path diff} = 2n \times \lambda/2}$ i.e even multiple of $\lambda/2$.

Maximum amplitude $\boxed{A_{max} = a+b}$

$\therefore I_{max} \propto (a+b)^2$

(ii) Condition of destructive interference

For destructive interference, intensity and hence amplitude is minimum,

i.e $A = minimum$.

i.e $\cos\phi = -1 = \cos(2n+1)\pi$

i.e $\phi = (2n+1)\pi$.

$\therefore \boxed{\text{Phase diff} = (2n+1)\pi}$

$\therefore \boxed{\text{Path diff.} = (2n+1)\lambda/2}$ odd multiple of $\lambda/2$.

Minimum amplitude; $\boxed{A_{min} = a \sim b}$.

$I_{min} \propto (a \sim b)^2$

Remember that — $\dfrac{I_{max}}{I_{min}} = \dfrac{(a+b)^2}{(a\sim b)^2}$

State the conditions of sustained / observable interference.

(V.U – 1998, 2001, 2003)

The conditions of sustained interference are —

i) The two superposing waves must be of same wavelength and preferably of same amplitude.

ii) They must come from two coherent sources.

iii) They should travel along the same direction.

iv) The two interfering waves must be in the same state of polarisation.

Explain — "Why light from two different candles are not seen interfere"?

When the light waves from two separate candles meet at a point, the point will be bright or dark according as they meet in same phase or opposite phases. It depends on two ~~terms~~ conditions —

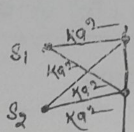
i) the path diff.

ii) phase relationship

Though path difference of the point from two sources is same for all times but phase relation are not constant for all times i.e the sources are not coherent. Thus they do not produce sustained inference.

Explain — "Light energy is conserved by interference".

Let us consider two coherent sources S_1 and S_2 produ one dark fringe and one bright fringe.. ~~Let~~ amplitude = a



dark $K \cdot 0^2 = 0$

bright $K \cdot (2a)^2 = 4Ka^2$

Intensity of four waves of two sources $= 4Ka^2$.

At the bright point, we get amplitude $= 2a$ ∴ intensity $= 4Ka^2$

At the dark point, we get amplitude $= 0$ ∴ intensity $= 0$.

Total intensity $= 4Ka^2$.

Classification of different methods for production of int. fringes.

There are two methods for production of interference fringes ——

i) Division of wavefront.

ii) Division of amplitude.

⟨A⟩ Division of wavefront ——

Wavefront is the locus of all points of the medium having same phase. Optical devices which divide the incident wavefront laterally into two parts by reflection or refraction and thereby give rise to two coherent sources, are under the division of wavefront class.

In order to maintain spatial coherence narrow source is necessary in this case.

Examples —— i) Formation of fringes by biprism.

ii) Formation of fringes by Lloyd's single mirror.

⟨B⟩ Division of amplitude ——

Optical devices which divide the amplitude of incident light wave into two or more parts by both partial reflection and refraction and thereby give rise to two or more i.e. coherent interfering beams, are under the division of amplitude class.

Broad source is necessary in this case.

Examples —— i) Formation of fringes by Newton's ring.

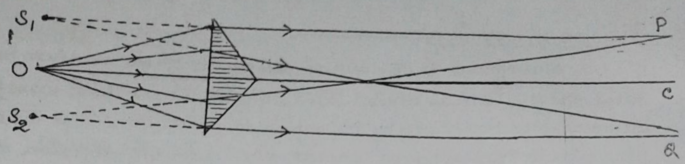
ii) Formation of fringes by thin film.

(B) **Describe the method of finding the wavelength of light by Fresnel's biprism. Deduce the theory in this case.**

(V.U - 1995, 1999, 2002)

FRESNEL'S BIPRISM

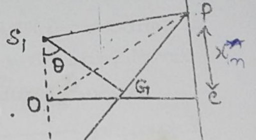
Biprism is nothing but two acute angled prisms placed base to base. In this case, fringes are produced based on the principle of division of wavefront.



Formation of fringes

Monochromatic light from a narrow source O is incident on the plane face of biprism, perpendicularly placed. Light from O is refracted by the two halves of the biprism, appear to diverge from the two virtual sources S_1 and S_2 which are therefore coherent. The waves from S_1 and S_2 will superpose on the screen within the region PCQ, where interference fringes are produced.

Theory



Let the distance between two virtual coherent sources $S_1S_2 = d$

Distance between source and screen $OC = D$.

Let nth order ~~bright~~ fringe is produced at P at a distance x_n from central fringe C.

Path difference at P $= S_2G_1$

From similar triangle $S_1S_2G_1$ and OPC

$$\frac{S_2G_1}{S_1G_1} = \frac{PC}{OC}$$

$$\Rightarrow S_2G_1 = \frac{PC}{OC} \times S_1G_1 = \frac{x_n}{D} \times d$$

Hence path diff. at P $= \frac{x_n d}{D}$.

i) For n^{th} order bright fringe at P,

path difference $= 2n \times \lambda/2$

$$\Rightarrow \frac{x_n d}{D} = n\lambda$$

$$\Rightarrow x_n = \frac{n\lambda D}{d}$$

For the next bright fringe, $x_{n+1} = \frac{(n+1)\lambda D}{d}$.

Thus distance between two successive bright fringes,

ie fringe width $\beta = x_{n+1} - x_n$.

$$\therefore \boxed{\beta = \frac{\lambda D}{d}}$$

ii) For n^{th} order dark fringe at P,

$$\text{Path diff.} = (2n+1)\lambda/2$$

$$\Rightarrow \frac{x_n d}{D} = (2n+1)\lambda/2$$

$$\Rightarrow x_n = \frac{(2n+1)\lambda D}{2d}$$

For the next dark fringe, $x_{n+1} = \frac{(2n+3)\lambda D}{2d}$.

Thus, distance between two successive dark fringes,

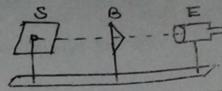
ie fringe width $\beta = x_{n+1} - x_n$.

$$\therefore \boxed{\beta = \frac{\lambda D}{d}}$$

Knowing the value of β, D and d we can compute λ.

Experimental arrangement:

The slit, biprism and the eyepiece setting on an optical bench, the slit is illuminated. Fringes are made visible by adjusting the eyepiece. By lateral movement of the prism stand and the eyepiece, the system is made co-axial.



i) For a fixed distance of the eyepiece from the slit the fringewidth (β) is measured with the help of eyepiece. The index marks on the ~~eyep~~ scale does not give the correct value of D. Hence index correction is necessary. Index error can be eliminated if we measure β for different position of eyepiece (i.e. D_1, D_2)

$$\therefore \beta_1 = \frac{\lambda(D_1+\alpha)}{d}$$

$$\beta_2 = \frac{\lambda(D_2+\alpha)}{d}$$

$$\therefore \beta_1 - \beta_2 = \frac{\lambda(D_1-D_2)}{d}$$

$$\therefore \lambda = \frac{(\beta_1-\beta_2)d}{D_1-D_2}.$$

ii) A convex lens of suitable focal length (f) is placed between the biprism and slit. When $D > 4f$ we get pair of images for two position of lens. One of them is magnified (d_1) and other is reduced (d_2)

Then distance between two sources $d = \sqrt{d_1 d_2}$

$$\therefore \boxed{\lambda = \frac{(\beta_1-\beta_2)\sqrt{d_1 d_2}}{D_1-D_2}}$$

Measuring d_1, d_2 and D_1, D_2, β_1, β_2, we can compute λ.

Measurement of the angle of the biprism.

OR **Show that the distance between two virtual sources in biprism experiment** $d = 2a(\mu-1)\alpha$

(V.U - 1997)

Deviation of each of the ray after passage through the two thin prisms $= \delta$ (radian)

Thus angular separation of the two virtual sources S_1 and S_2 $= 2\delta$.

But for small angled (α) prism, deviation $\delta = (\mu-1)\alpha$

Thus $2\delta = 2(\mu-1)\alpha$.

Again

If $a \Rightarrow$ distance between slit and the biprism.
$d \Rightarrow$ distance between two virtual sources.

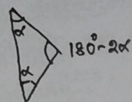
Then $2\delta = \frac{d}{a}$ $\left(\text{angle} = \frac{\text{arc}}{\text{radius}}\right)$.

$\therefore \quad 2(\mu-1)\alpha = \frac{d}{a}$.

$\therefore \quad \boxed{\alpha = \frac{d}{2(\mu-1)a}}$ Acute angle of biprism in radian unit.

Hence $\boxed{d = 2a(\mu-1)\alpha}$

* Obtuse angle of the biprism $= 180° - 2\alpha$.
calculate α in degree.



Why the base angle of a biprism usually very small? (V.U -1997)

$d = 2a(\mu-1)\alpha$.

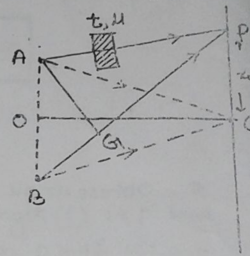
When α is made large then the distance between two coherent sources (d) becomes large and hence fringe width $\beta = \frac{\lambda D}{d}$ becomes small. Thus the fringes can't be distinguished. Hence the base angle of a biprism usually very small.

(12) Why the plane surface of the biprism is placed towards the biprism ?

If the plane surface of the biprism is taken in front of the biprism then the deviation of refracted light will be minimum. Thus the virtu distance between the virtual sources (d) will be minimum. Thus fringe width $\beta = \frac{\lambda D}{d}$ will be large and the fringes can be distinguished.

(13) Measurement of Thickness of a thin film or mica by interference method :-

Let A and B are two mono-chromatic coherent sources which are producing interference pattern on the screen.



Since AC and BC are equal path thus the central bright fringe will produced at C.

Let a thin film of thickness t and r.i. μ be introduced in the front of any source, so that central fringe is shifted to P at a distance x_0 from C.

Thus path optical path difference at P is zero i.e.

Optical path AP = Optical path BP.

i.e. $\mu t + (AP - t) = BP$

$\Rightarrow \quad BP - AP = (\mu - 1)t$

$\therefore \quad BG = (\mu - 1)t$ ——— (1)

Let the distance between two coherent sources $= d$ and distance between source and screen $= D$.

Then $BG = \frac{x_0 d}{D}$ ——— (2)

From equation (1) and (2), we get ———

$$(\mu-1)t = \frac{x_0 d}{D}$$

$$\Rightarrow \boxed{t = \frac{x_0 d}{(\mu-1)D}}$$

Corollary —— (1) If fringe width β is given,

Then $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \frac{d}{D} = \lambda/\beta$$

Then Thickness of film $\boxed{t = \frac{x_0 \lambda}{(\mu-1)\beta}}$

Corollary —— (2) If no. of fringe shift is given (m)

Then $m = \frac{x_0}{\beta}$

Then Thickness of film, $\boxed{t = \frac{m\lambda}{\mu-1}}$

Problem (1)
V.U – '99

Two coherent sources separated by a distance of 0.6 mm. produce interference pattern on a screen placed 1 m. away from the sources. The wavelength of the light used is 6000 A°. Find the fringe-width.

If a thin glass plate $(\mu = 1.5)$ is placed in front of one of the sources, the central bright fringe in this arrangement coincides with the 3rd dark fringe of the previous one. Find the thickness of the glass plate.

Solution —

i) $d = 0.6\ mm = 0.06\ cm$.

$\lambda = 6000\ A^\circ = 6000 \times 10^{-8}\ cm$.

$D = 1\ m = 100\ cm$.

$\therefore$ Fringe width $\beta = \dfrac{\lambda D}{d}$

$= \dfrac{6 \times 10^{-5} \times 100}{0.06}$

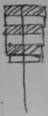
$= \dfrac{6 \times 10^{-3} \times 100}{6}$

$= \underline{0.1\ cm}$.

ii)

Distance of thick dark fringe from c,

$x_3 = \dfrac{(2\times3+1)\ \lambda D}{2d}$



Thus shifting distance $x_0 = \frac{7}{2} \times 0.1\ cm$.

$= 0.35\ cm$.

Thickness of glass plate $t = \dfrac{x_0 d}{(\mu-1) D}$

$= \dfrac{0.35 \times 0.06}{(1.5-1) \times 100}$

$= \dfrac{35 \times 6 \times 10^{-4}}{5 \times 10}$

$= 42 \times 10^{-4}\ cm$.

$= \underline{0.0042\ cm}$

Problem (2)
V.U – 2001

Find the maximum no. of fringes formed by a biprism of r.i. 1.5 and vertical angle 1° when it is placed at a distance of 20 cm. from the monochromatic narrow source of light having wavelength 6000 A°. The fringes are observed on a screen 80 cm. away from the biprism.

Deduce the formula to be used.

Why the above no. of fringes to be taken as maximum

(19)

Solution ———

$\mu = 1.5$

$\alpha = 1^\circ = \frac{\pi}{180}$ radian.

$a = 20$ cm.

$\lambda = 6000\,A^\circ = 6000 \times 10^{-8}$ cm.

$D = \underset{(a+b)}{(20+80)}$ cm $= 100$ cm.

Distance between virtual sources $d = 2a(\mu-1)\alpha$.

$= 2\times20\times0.5\times\frac{\pi}{180}$ cm

$= \frac{20\times3.14}{180}$ cm

$= \frac{3.14}{9}$ cm.

Fringe width $\beta = \frac{\lambda D}{d}$

$= \frac{6\times10^{-5}\times100\times9}{3.14}$ cm.

$= \frac{\lambda(a+b)}{2a(\mu-1)\alpha}$.

Overlapping region. $AB = b\cdot2\delta$

$= b\cdot 2(\mu-1)\alpha$

$= 2b(\mu-1)\alpha$

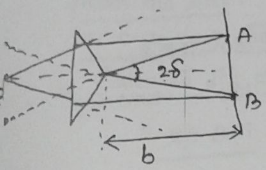
Thus maximum no. of observable fringe,

$$m = \frac{AB}{\beta} = \frac{2b(\mu-1)\alpha\times2a(\mu-1)\alpha}{\lambda(a+b)}$$

$$= \frac{4ab(\mu-1)^2\alpha^2}{\lambda(a+b)}$$

$$= \frac{4\times20\times80(1.5-1)^2\times\left(\frac{\pi}{180}\right)^2}{6\times10^{-5}(20+80)}.$$

$= \ldots\ldots$

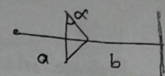


Problem (3)
V.U - 1995

When a thin monochromatic source of light was placed at a distance of 50 cm. from a Fresnel's biprism of $\mu = 1.5$. The distance between two consecutive bands formed on a screen placed at a distance of 100 cm. from biprism was found to be 0.012 cm. If the wavelength of the light was 5893×10^{-8} cm. find the obtuse angle of the biprism.

Solution

$a = 50$ cm.
$\mu = 1.5$
$\beta = 0.012$ cm.
$b = 100$ cm.
$\lambda = 5893 \times 10^{-8}$ cm.
$\therefore D = a + b = 150$ cm.



Acute angle of biprism $\alpha = \dfrac{d}{2a(\mu-1)}$ again $\beta = \dfrac{\lambda D}{d}$

$\Rightarrow d = \dfrac{\lambda D}{\beta}$.

$$\therefore \alpha = \frac{\lambda D}{2a(\mu-1)\beta} \text{ (in radian)}$$

$$= \frac{5893 \times 10^{-8} \times 150}{2 \times 50 (1.5 - 1) \times 0.012}$$

$$= 0.015 \text{ radian}$$

$$= 0.015 \times \cancel{\frac{\pi}{180}} \frac{180}{\pi} \text{ degree}$$

$$= 0.84 \text{ degree}$$

$\therefore$ Obtuse angle of the biprism $= 180^\circ - 2\alpha$

$$= 180^\circ - 2 \times 0.84^\circ$$

$$= 178.32^\circ$$

Problem (4)
C.U – 2000

Ans.

Fringes are produced with monochromatic light of wavelength 589 nm. A thin film of glass of r.i. 1.52 is placed normally in the path of one of the interfering rays. The central bright fringe is found to move to a position occupied by the fifth bright band from the centre. Calculate the thickness of the glass film.

$$\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}.$$
$$\mu = 1.52.$$
$$m = 5$$

Thickness $t = \dfrac{x_0 d}{(\mu-1) D}$. here $x_0 = m\dfrac{\lambda D}{d}$.

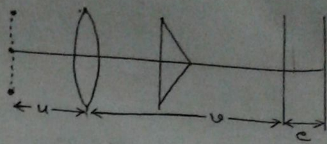
$\therefore \dfrac{d}{D} = \dfrac{5\lambda}{x_0}$.

$$= \frac{m\lambda \cancel{D} \cancel{d}}{(\mu-1) \cancel{D} \cdot \cancel{d}}$$
$$= \frac{m\lambda}{\mu - 1}.$$
$$= \frac{5 \times 589 \times 10^{-9}}{1.5 - 1}$$
$$= 6.625 \times 10^{-6} \text{ m}.$$

Problem (5).
V.U – 2004

In an experiment with Fresnel's biprism bands of width 0.0196 cm are observed at a distance of 100 cm. from slit. A convex lens is then introduced between the screen and the biprism so as to give an image of the sources on the screen 100 cm. apart from the slit. The distance between the images is found to be 0.70 cm, the lens being 30 cm. from the slit.
Calculate the wavelength of light used.

Solution :



Given, $\beta = 0.0196$ cm.

$D = 100$ cm.

$u = 30$ cm.

$v = 70$ cm.

$c = 0.70$ cm.

According to figure, $\frac{v}{u} = \frac{c}{d}$

$$\Rightarrow d = \frac{u}{v} \times c = \frac{30}{70} \times 0.70 = 0.3 \text{ cm.}$$

$\therefore$ Fringe width $\beta = \frac{\lambda D}{d}$.

Thus, $\lambda = \frac{\beta d}{D} = \frac{0.0196 \times 0.3}{100}$

$$= 5.88 \times 10^{-5} \text{ cm.}$$

Problem (6)

V.U - 2008

Given, $D = 200$ cm.

$d = 0.05$ cm. <diameter of the wire>.

$\lambda = 690 \times 10^{-7}$ cm.

$\therefore$ Fringe width $\beta = \frac{\lambda D}{d}$

$$= \frac{69 \times 10^{-6} \times 200}{0.05}$$

$$= \frac{138}{5} \times 10^{-2} \text{ cm}$$

$$= 0.276 \text{ cm.}$$