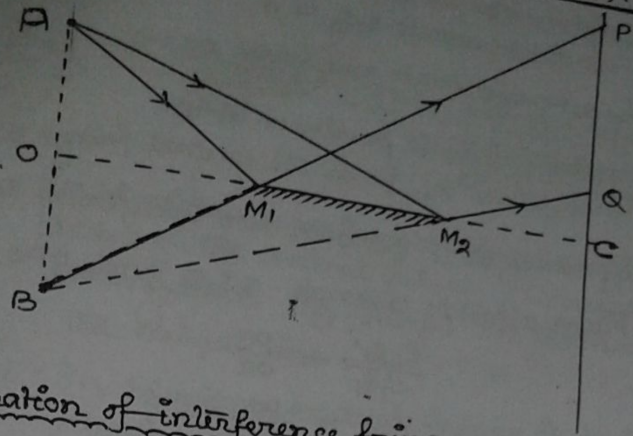
(18) Lloyd's single mirror

How does the wavelength of monochromatic light is measured with the help of Lloyd's mirror. Deduce the formula used (V.U – 1994, 1997, 2000, 2003)



Formation of interference fringe :—

In Lloyd's single mirror arrangement a small metallic mirror is fitted whose reflecting surface is plane and highly polished. The light wave from a narrow source (A) of monochromatic light when incident on the mirror M_1M_2, the reflected lights incident on the screen, which give rise to a virtual source B. The real source A and virtual source B behaves like coherent source. Direct waves from source A and the reflected waves from B on superposition at the region PQ will give interference fringes.

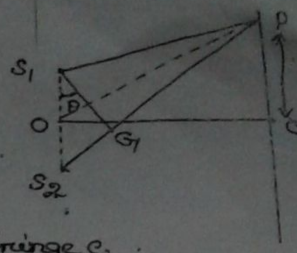
Characteristics of fringes in Lloyd's mirror —

i) Here the central fringe is dark.
ii) Here the fringes are one sided.
iii) A phase change of π is created due to reflection of light from denser to rarer medium.
iv) Wave front is devided by the method of division of wavefront.

Theory

Let the distance between two coherent sources $S_1S_2 = d$

Distance between source and screen $OC = D$.



Let nth order fringe is produced at P at a distance x_n from the central fringe C.

Path difference at P between two interfering beam $= S_2G$.

From similar triangle S_1S_2G and OPC,

$$\frac{S_2G}{S_1G} = \frac{PC}{OC}$$

$$\Rightarrow S_2G = \frac{PC}{OC} \times S_1G$$

$$= \frac{PC}{OC} \times S_1S_2$$

$$= \frac{x_n d}{D}$$

(since G is very small near to ...)

Total path difference = path diff. due to different path + path diff. due to reflection

$$= \frac{x_n d}{D} \pm \lambda/2 .$$

i) For nth order bright fringe at P.

$$\text{Path diff.} = 2n \times \lambda/2 .$$

$$\therefore \frac{x_n d}{D} \pm \lambda/2 = 2n \times \lambda/2$$

$$\Rightarrow x_n = \frac{(2n+1)\lambda D}{2d} ,$$

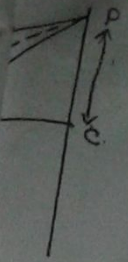
For the next bright fringe $x_{n+1} = \frac{(2n+3)\lambda D}{2d}$.

$\therefore$ Fringe width $\beta = x_{n+1} - x_n = \frac{\lambda D}{d}$.

(ii) For nth order dark fringe at P.

$$\frac{x_n d}{D} \pm \lambda/2 = (2n+1)\lambda/2$$

$$\therefore x_n = \frac{n\lambda D}{d} .$$



Fringe width $\beta = \frac{\lambda D}{d}$

Measuring β, d and D, we can compute λ. Ans.

Experimental arrangement —

As like biprism experiment.

What is the difference between biprism and Lloyd's fringes?

(V.U - 1994, 1997, 2000)

* i) In biprism experiment fringes are formed on both sides of the central fringe., Whereas in Lloyd's mirror arrangement the fringes are formed on one side of the central line.
* ii) In biprism central fringe is bright whereas in Lloyd's mirror central fringe is dark.
* iii) In biprism the separation between every pair of corresponding points (d) of the coherent sources is same, hence fringe width is same for all parts of the source.

  But in Lloyd's mirror due to lateral inversion of the mirror ~~is~~, d is different for different pair of corresponding points, hence fringe width is not same for all parts of the source.
* iv) In biprism wavefront is devided by refraction hence there is no additional phase difference.

  But in Lloyd's mirror wavefront is devided by reflection of light hence there is an addition phase difference $\pm\pi$.
* v) In biprism, distance of n th bright ~~from~~ fringe from c $x_n = \frac{n\lambda D}{d}$.

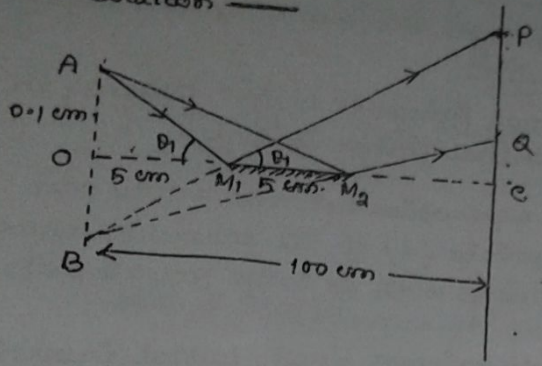
  In Lloyd's mirror. That is, $x_n = \frac{(2n+1)}{2}\frac{\lambda D}{d}$.

(2)

(B) Problem (I) V.U-1997 (B) A Lloyd's mirror of length 5 cm is ...

(22)

Solution —



Given —

$M_1M_2 = 5$ cm.

$\lambda = 6000 \times 10^{-8}$ cm.

$OA = \frac{d}{2} = 0.1$ cm.

$\therefore d = 0.2$ cm.

$OM_1 = 5$ cm.

$D = 100$ cm.

i) Fringe width $\beta = \frac{\lambda D}{d} = \frac{6000 \times 10^{-8} \times 100}{0.2}$ cm

$= \underline{0.03 \text{ cm}}$.

ii) Width of the fringe system $= PQ = CP - CQ$.

From similar ΔM_1CP and ΔM_1OA.

$\tan\theta_1 = \frac{CP}{M_1C} = \frac{OA}{OM_1}$

$\Rightarrow CP = \frac{OA}{OM_1} \times M_1C$ since $M_1C = 100 - 5$ cm $= 95$ cm.

$= \frac{0.1 \times 95}{5}$

$= 1.9$ cm.

From similar triange $\triangle AOM_2$ and $\triangle M_2QC$,

$$\tan\theta_2 = \frac{QC}{M_2C} = \frac{OA}{OM_2}$$

$$\Rightarrow CQ = \frac{OA}{OM_1} \times M_2C$$

$$= \frac{0.1 \times 90}{10}$$

$$= 0.9 \text{ cm.}$$

since $M_2C = 100 - (5+5)$ cm $= 90$ cm.

Thus width of the fringe system $= (1.9 - 0.9)$ cm.
$= \underline{1 \text{ cm}}$

(ii) Let the amplitude of incident ray $= a$.
and amplitude of reflected ray $= b$

Then,

$$\frac{\text{intensity of incident ray}}{\text{intensity of reflected ray}} = \frac{a^2}{b^2}$$

$$\Rightarrow \frac{100}{81} = \frac{a^2}{b^2}$$

$$\Rightarrow \frac{b}{a} = \frac{9}{10}$$

$$\Rightarrow b = \frac{9}{10}a.$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a\sim b)^2} = \frac{\left(a + \frac{9}{10}a\right)^2}{\left(a - \frac{9}{10}a\right)^2}$$

$$= \frac{(19/10)^2}{(1/10)^2} = 19 \times 19 = \underline{\underline{361}}.$$

Problem (2)
V.U − 2000

A Lloyd's mirror experiment is performed with a microwave source emitting waves of wavelength 40 cm. Using a plane metal sheet on a table top as the reflector. If the source is placed 6 cm. above the table; find the height of the 1st maxima above this surface at a distance of 400 cm. from the source.

Solution —

In Lloyd's mirror,
condition of nth order bright fringe,

$$\text{path diff} = 2n \times \lambda/2 \pm \lambda/2$$

~~For 1st maxi~~

$$\frac{x_n d}{D} = (2n-1)\lambda/2$$

$$\Rightarrow x_n = (2n-1)\frac{\lambda D}{2d}$$

for 1st maxima, $x_1 = \frac{\lambda D}{2d}$

Given —

$\lambda = 40$ cm

$d/2 = 6$ cm $\therefore d = 12$ cm

$D = 400$ cm

$$x_1 = \frac{40 \times 400}{2 \times 12} = \frac{2000}{3} \text{ cm} = 666.67 \text{ cm}$$