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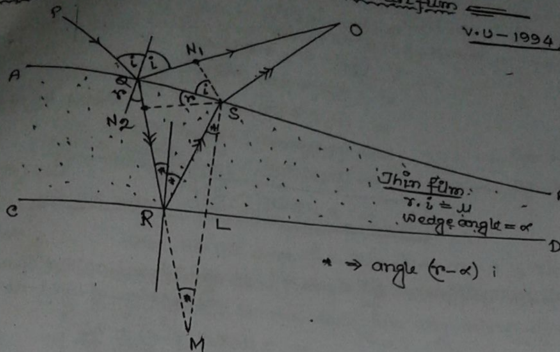
Thin film and Newton's ring

Explain the interference phenomena in thin film

V.U – 1994, 2000



* → angle $(r-\alpha)$;

Formation of fringes

We consider a thin wedge-shaped film bounded by surfaces AB and CD, having r.i. = μ and wedge angle = α.

A plane wave of monochromatic light (wavelength λ) incident on it along PQ. It is partially reflected along QO and partially refracted along QR. The refracted wave is again reflected at R along RS and finally refracted along SO. These two emergent light along QO and SO on superposition gives interference fringes.

Theory

Let the thickness of the film at S ; SL = e.

From figure, $\triangle$RMS is bilateral ; Thus RS = RM.

and SL = LM

$\therefore$ SM = 2e.

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Path difference between two interfering rays at O,

$$
\begin{aligned}
l &= (QR + RS)\mu - QN_2 \\
&= (QR + RS)\mu - QN_2 \times \mu \\
&= (QR - QN_2 + RS)\mu \\
&= (N_2R + RS)\mu \\
&= (N_2R + RM)\mu \\
&= \cancel{N_2R \times \mu} \\
&= N_2M \times \mu
\end{aligned}
$$

$$\boxed{l = 2\mu e \cos(r-\alpha)}$$

applying Snell's law at Q;

$\mu = \frac{\sin i}{\sin r} = \frac{QN_1}{QN}$

$\therefore QN_1 = \mu \times QN$

Since $RS = RM$.

From $\triangle SN_2M$,

$\cos(r-\alpha) = \frac{N_2M}{SM}$

$\Rightarrow N_2M = 2e\cos(r-\alpha)$

as $SM = 2e$

Total path difference at O $= l \pm$ additional path difference due to reflection of light at Q

$= 2\mu e \cos(r-\alpha) \pm \lambda/2$

Condition of maxima —

For nth order maxima at O,

path difference = even multiple of $\lambda/2$

$\Rightarrow 2\mu e\cos(r-\alpha) \pm \lambda/2$ = even multiple of $\lambda/2$

$\Rightarrow \boxed{2\mu e\cos(r-\alpha) = (2n+1)\lambda/2}$

Where $n = 0, 1, 2 \ldots$

Condition of minima —

For nth order minima at O,

path difference = odd $(2n\pm1)$ multiple of $\lambda/2$

$\Rightarrow 2\mu e\cos(r-\alpha) \pm \lambda/2$ = odd multiple of $\lambda/2$

$\Rightarrow \boxed{2\mu e\cos(r-\alpha) = 2n \times \lambda/2}$

Where $n = 0, 1, 2, \ldots$

Discussion

i) **When the thin film is parallel**

Then $\alpha = 0$ i.e. $\cos(r-\alpha) = \cos r$

For bright fringe (nth order) at O, $2\mu e \cos r = (2n+1)\,\lambda/2$.

For dark fringe (nth order) at O, $2\mu e \cos r = 2n \cdot \lambda/2$.

ii) **For normal incidence**

Then $i = r = 0$

For bright fringe (nth order) at O, $2\mu e = (2n+1) \cdot \lambda/2$.

For dark fringe (nth order) at O, $2\mu e = 2n \cdot \lambda/2$.

iii) **At the edge of the thin film**

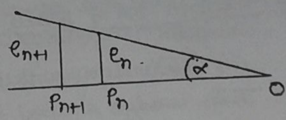
At the edge of a thin film $e = 0$; which satisfy the condition of dark fringe [i.e. $2\mu e \cos(r-\alpha) = 2n \cdot \lambda/2$]. Thus the fringe will be dark for any colour of light (λ).

iv) **If the film is extremely thin**

If the film is extremely thin then $e \to 0$; which satisfy the condition of dark fringes. Thus fringes will be dark for all over the film.

Calculation of fringe width in thin film

Let us consider a wedge-shaped film having wedge angle α.



Let nth order bright fringe is produced at P_n where the thickness of the film is e_n and the next $(n+1)$ order bright fringe is produced at P_{n+1} where the thickness of the film is e_{n+1}.

For normal incidence $(i=r=0)$ and small wedge angle $(\alpha=0)$
condition of nth order bright fringe, $2\mu e_n = (2n+1)\lambda/2$
condition of $(n+1)$th order bright fringe, $2\mu e_{n+1} = (2n+3)\lambda/2$

Substracting, $2\mu[e_{n+1} - e_n] = \lambda$

$$\Rightarrow e_{n+1} - e_n = \frac{\lambda}{2\mu} \quad \text{——— (1)}$$

From two similar triangle,

$$\tan\alpha = \frac{e_n}{OP_n} \quad \therefore OP_n = \frac{e_n}{\tan\alpha}$$

$$\tan\alpha = \frac{e_{n+1}}{OP_{n+1}} \quad \therefore OP_{n+1} = \frac{e_{n+1}}{\tan\alpha}$$

Fringe width, $\beta = P_nP_{n+1} = OP_{n+1} - OP_n$

$$= \frac{e_{n+1} - e_n}{\tan\alpha} \quad \text{from eq}$$

$$= \frac{\lambda}{2\mu\tan\alpha}$$

$$= \frac{\lambda}{2\mu\alpha} \quad \text{since } \alpha \text{ is small}$$

$$\therefore \boxed{\beta = \frac{\lambda}{2\mu\alpha}}$$

(2) **Fringes with white light : Colours of thin film.** V.U – 2001

When a parallel beam of white light is incident on thin wedge shaped film, the values of n, λ and r will be different for different colours of light.

At the edge of the thin film $e \to 0$; which satisfy the condition of dark for any value of λ and r; thus edge will be perfectly dark for all colours of light.

— For white light, as $\lambda_r > \lambda_v$ thus fringe ~~width~~ for violet ray will be formed at a smaller thickness of the film while the corresponding bright fringe of red light will be formed at the greater thickness. (1st order)

since $\beta = \frac{\lambda}{2\mu\alpha}$.

$$\therefore \beta_r = \frac{\lambda_r}{2\mu\alpha}.$$

$$\beta_v = \frac{\lambda_v}{2\mu\alpha}.$$

since $\lambda_r > \lambda_v \therefore \beta_r > \beta_v$.

(B) Fringes with equal width and equal inclination —

For a thin wedge shaped film ——

condition of maxima, $2\mu e \cos(r-\alpha) = (2n+1)\lambda/2$.

condition of minima, $2\mu e \cos(r-\alpha) = 2n \cdot \lambda/2$.

Fringes of equal width —— (Fizeau's fringe).

When for a plane parallel beam of monochromatic light is incident on a thin film. λ, μ, r are constant. Hence different order (n) of fringes will be controlled by the thickness (e) of the film. Hence a fringe of particular order number will lie on the locus of all the points of the film having a constant thickness. These fringes are called fringes of equal width or thickness.

Example —— Newton's ring.

Fringes of equal inclination — (Haidinger's fringe)

When monochromatic divergent light from a source incident on a plane parallel thin film then λ, μ and e constant. But order number (n) varies with r thus hence angle of incidence. Hence a particular order of fringe will be produce for particular angle of incidence. These fringes are called fringes of equal incidence.

Example — Michelson's interferometer fringe

Problem (1) V.U - 2008

An air wedge is formed between two glass plates. It is found that the interference fringes are 1.00 mm. apart when the air wedge is illuminated normally with a light wavelength 589.3 nm. Find the angle of wedge?

Solution: Given $\beta = 1\ mm. = 0.1\ cm.$

$\lambda = 589.3\ nm. = 589.3 \times 10^{-7}\ cm.$

$\mu = 1$ (air).

We know, fringe width in terms of wedge angle

$$\beta = \frac{\lambda}{2\mu\alpha}.$$

$$\therefore \alpha = \frac{\lambda}{2\mu\beta} = \frac{589.3 \times 10^{-7}}{2 \times 1 \times 0.1} = 2946.5 \times 10^{-9}\ \text{radian}.$$

Problem (2) V.U - 2009

A Thin wedge of air is formed between two plane glass plates. A parallel beam of light of wavelength 5000 A^0 is incident normally on it. If there are 200 fringes per cm, calculate the wedge angle.

Given, $\lambda = 5000\ A^0 = 5000 \times 10^{-8}\ cm.$

$\beta = \frac{1}{200}\ cm.$

$\mu = 1$ (air).

We know, fringe width $\beta = \frac{\lambda}{2\mu\alpha}$,

$$\therefore \alpha = \frac{\lambda}{2\mu\beta} = \frac{5000 \times 200}{2 \times 1 \times 1}$$

$$\therefore \underline{\alpha = 0.005\ \text{radian}}\ \text{Ans.}$$