compensating plate ? (2003)

fringes with white light because optical path depends on wavelength.

(1)

☒ Explain how fringes are formed in Michelson's Interferometer

V.U – '98, '2003

or Under what circumstances straight and circular fringes are formed ? V.U – '95, 2001

Form of fringes ——

On looking along the direction TO, we see that the mirror M_1 and image M_2' of mirror M_2 formed by reflection from the semi silvered surface of P_1

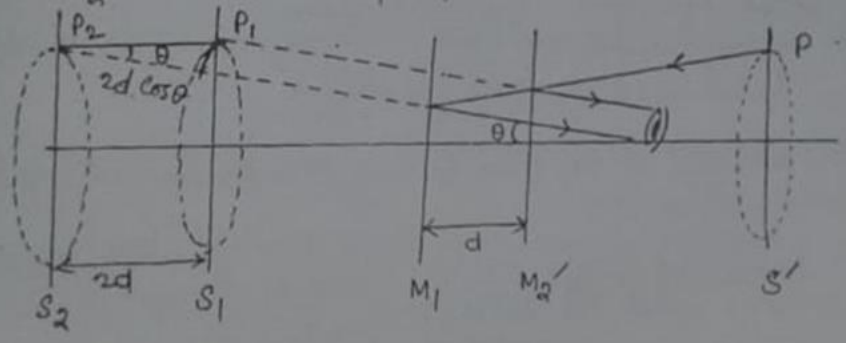
Thus we may consider that one of interfering beam comes by reflection from M_1 and the other appears to come by reflection from M_2'. Depending on the separation (d) between M_1 and M_2' and angle between surfaces we may get fringes of different shape - straight, circular etc.

1) Formation of circular fringes ——

When M_2 is exactly perpendicular to M_1, the film M_1M_2' is of uniform thickness and we get circular fringes as explained below ——

M_1 and M_2' are parallel reflecting surfaces. The actual source has been replaced by its virtual source S' formed by reflection in the partially silvered surface. S' forms two virtual images S_1 and S_2 in M_1 and M_2'.

Light from a point P on the extended source appears to an eye to come from the corresponding points P_1 and P_2 on S_1 and S_2.



If d be the distance between M_1 and M_2' then the distance between S_1 and $S_2 = 2d$. If θ be the inclination of the reflected ray with the normal; then path diff between the two rays $= 2d\cos\theta$

For nth order bright fringe, $2d\cos\theta = n\lambda$

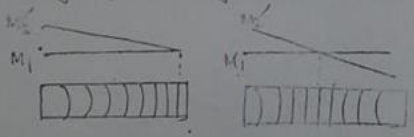
For nth order dark fringe, $2d\cos\theta = (2n+1)\lambda/2$.

Since d and λ is constant, we get a no. of concentric bright circular fringes for different values of D.

Nature of fringes — i) Concentric circular fringes.
ii) Since the interfering rays are parallel these fringes are formed at infinity.
iii) Central ring is dark.
iv) They are non-localised fringes.
v) The fringes are called fringes with equal inclination.

ii) Formation of straight fringes —

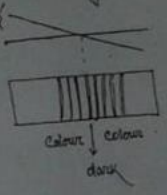
When M_2 is not perpendicular to M_1, the air film between M_1 and M_2 is wedge-shaped. Since light is incident on the film at different angles we, curved fringes with convexity towards the thin edge of the wedge are obtained.



If the thickness of the film is very small, the fringes are practically straight.

Nature of fringes — i) Straight fringes.
ii) Localised fringes.

being again perfectly achromatic and st... light, fringes will be observed only when... not exceed a few wavelengths.



What are localised and non-loca...

Localised fringes — Localised fri... surface a is wedge... of the screen. Example —

Non-localised fringes — Non-loc... ted to any surfa... Example

What are Fizeau and Haid...

The fringes of equal wid...
The fringes of equal incl... fringes

4. Standardisation of

Determination of wavelength of monochromatic light using Michelson's Interferometer

The Interferometer is set for circular fringes i.e M_1 is made perpendicular to M_2. Position of the mirror M_1 is adjusted to obtain a bright spot at the centre of the field of view.

If d be the Thickness of the film, for n^{th} order bright fringe, $2d\cos\theta = n\lambda$.

here $\theta = 0$ $\therefore 2d = n\lambda$ ——— (1)

If now the mirror M_1 is moved away from M_2 by $\lambda/2$ then $2d$ increases by λ. Therefore $(n+1)$ replaces n in eqn. (1). Hence $(n+1)^{th}$ order bright spot will produce at the centre.

Suppose during the movement of M_1 through a distance x, N new fringes appear at the centre.

Then $x = N \cdot \lambda/2$.

$$\Rightarrow \boxed{\lambda = \frac{2x}{N}} \quad \text{——— (2)}$$

Measuring the distance x on the micrometer screw and counting the number N (no. of change of bright fring at the centre), the value of λ can be obtained by equation (2).

Determination of difference in wavelengths by Michelson's Interferometer

(5)

OR Show how to study sodium doublet with the help of Michelson's Interferometer. V.U – 1998

OR How can Michelson's Interferometer be used to measure the wavelength difference between Na D_1 and D_2 lines? V.U – 2001

If the source emits light of two wavelengths λ_1 and λ_2 ($\lambda_1 > \lambda_2$) then each wave will produce an interference system of its own. If the mirror M_1 is displaced, the field will be alternately distinct and indistinct.

The field will be distinct i.e the fringes will be consonance when the bright ring of one wave coincide with the bright ring of another wave. The field will be indistinct i.e the fringes will be indistinct i.e the fringes will be dissonance when the bright ring of one wave coincide with the dark ring of another.

Let the mirror M_1 is displaced by a distance d. So that the fringes pass from one consonance to the next consonance,

Then we may write, $d = \frac{n\lambda_1}{2}$ i.e $2d = n\lambda_1$

and $d = \frac{(n+1)\lambda_2}{2}$ i.e $2d = (n+1)\lambda_2$

or $\frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = 1$

$\Rightarrow 2d \times \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} = 1$

$\Rightarrow \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2d}$

Since λ_1 and λ_2 are closer, thus $\lambda_1 \lambda_2 = \lambda^2$, Where λ is the mean of λ_1 and λ_2.

$$\therefore \boxed{\lambda_1 - \lambda_2 = \frac{\lambda^2}{2d}}$$

Measuring d and λ, we can compute $\lambda_1 - \lambda_2$.

N.B. If the source is Na (sodium), then it emits two wavelengths as Na D_1 line and Na D_2 line. By the above process, we can compute $\lambda_1 - \lambda_2$

☑ Determination of Thickness of Thin transparent sheet by a Michelson's Interferometer — V.U-'96,'97

☑ Determination of refractive index of a material.

To determine the r.i. or Thickness of a material (transp
the interferometer is to be first adjusted for white light fringes
The cross-wire is set on the achromatic fringe which is
perfectly straight.

The plate (mica) is now placed inserted in the pat
of one of the interfering beams.

This increases the optical path is $(\mu-1)t$

Where $\mu \Rightarrow$ r.i. of the material
$t \Rightarrow$ Thickness of the plate.

Since the beam traverses the plate twice, an extra path diff is produced between two interfering beams $= 2(\mu-1)t$.
Due to this extra path, the central fringe will be displaced from the wire.

The mirror M_2 is displaced by a distance d until the central fring is again coincides with the wire.

$$2d = 2(\mu-1)t$$

$$\Rightarrow \boxed{d = (\mu-1)t}$$

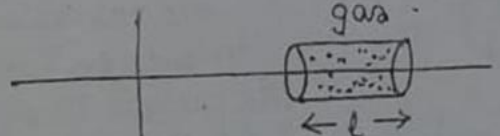
i) $\therefore \boxed{t = \frac{d}{\mu - 1}}$

Knowing μ and computing d, we can calculate ~~t~~ ie Thickness of the material.

ii) $\boxed{\mu = \frac{d}{t} + 1}$

Knowing t and computing d, we can calculate μ, ie the r.i. of the material.

N.B ⇒ Refractive index of gas —



$\boxed{d = (\mu - 1)\, l}$

What is etalon? How it is used for standardisation of metre?
V.U - 1998, 2003

Problem (1) A Thin film of oil is placed in the arm of a Michelson's Interferometer. If a shift of 7.5 fringes occour in the light of wavelength 5890 A°. What is the thickness of the film?

V.U – 1998

Solution — If t be the thickness of the film.
Additional path diff $= 2(\mu-1)t$.
If N be the no. of fringe shifted.
Then $2(\mu-1)t = N\lambda$.

$$\Rightarrow t = \frac{N\lambda}{2(\mu-1)}$$

Here, $N = 7.5$
$\lambda = 5890\ A^\circ$
$\mu = \cdots\cdots$
$t = ?$

Problem (2) The scale attached to the movable mirror of a Michelson's Interferometer records the mirror has traversed through 0.293 cm. for two successive coincidances. Calculate the wave number separation between the two lines of the given source. $\lambda = 5893\ A^\circ$.

Solution — If d be the distance traversed by the mirror,
Then $\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2d} = \frac{\lambda^2}{2d}$.

or, $\frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = 1$

$\Rightarrow \lambda_1 - \lambda_2 = \frac{\lambda^2}{2d}$

Here $\lambda = 5893\ A^\circ = 5893 \times 10^{-10}\ m$.
$d = 0.298\ cm = 298 \times 10^{-5}\ m$.

$$\therefore \quad \lambda_1 - \lambda_2 = \frac{(5893 \times 10^{-10})^2}{2 \times 2 \cdot 3 \times 10^{-5}} \text{ m}$$

$$= \frac{5893 \times 5893}{58.6} \times 10^{-15} \text{ m}.$$

Wavelength separation $\lambda_1 - \lambda_2 =$ ..... A^0.

Wave no. separation $= \frac{\lambda_1}{c} - \frac{\lambda_2}{c}$

$$= \frac{\lambda_1 - \lambda_2}{c} = \text{.....} \ A^0$$

Problem: In Michelson's interferometer one of the mirrors is moved slowly by a screw attached to it. The scale attached to the screw gives reading as follows;
10.73 mm for 10597 th fringe
10.7051 mm for 10497 th fringe.
Calculate the wavelength of light used. V.U – 1995

Given, $d = (10.73 - 10.7051)$ mm

$= 0.0249$ mm

$= 0.0249 \times 10^{-3}$ m.

Fringe shift $N = 10597 - 10497 = 100$.

$2d = N\lambda$

$\Rightarrow \lambda = \frac{2d}{N} = \frac{2 \times 249 \times 10^{-7}}{100}$ m.

$= 498 \times 10^{-9}$ m.

$= 498 \times 10^{-9} \times 10^{10} \ A^0$.

$= \underline{4980 \ A^0}$.

Problem. A Thin film with r.i. 1.4 for light of wavelength 5890 A° is placed in one arm of a Michelson's Interferometer. If a shift 7 fringes occours, what is the film Thickness ?

V.U – 1994.

Given —

$\mu = 1.4$

$\lambda = 5890\ A^0$

$N = 7$

$t = ?$

$2(\mu - 1)t = N\lambda$

$\therefore t = \frac{N\lambda}{2(\mu-1)} = \frac{7 \times 5890}{2(1.4-1)} = \frac{7 \times 5890}{2 \times 0.4}\ A^0$

$= \frac{7 \times 58900}{8}\ A^0$

$= ------\ A^0$

Problem. If one arm of Michelson's Interferometer contains a tube 2.5 cm. long which is first evacuated and Then filled with air ($n = 1.0003$), how many fringes will cross the centre ? ($\lambda = 600$ nm)

V.U – 1993

$l = 2.5$ cm.

$\mu = 1.0003$.

$\lambda = 600$ nm.

$N = ?$

$2d = (\mu - 1)l$.

again $2d = N\lambda$.

$\therefore N\lambda = (\mu - 1)l$

$\Rightarrow N = \frac{(\mu-1)l}{\lambda}$.

$= ....$

Coherence

(18)

Coherence :— The term coherence refers to the degree of co-relation between the phases at different points in a beam of light.

Coherence is of two types — ① Temporal coherence
② Spatial coherence.

Define and discuss "Temporal coherence". Also define coherence time

Temporal coherence :—

A beam of light said to posses temporal coherence if the phase difference of the wave crossing the two points lying along the direction of propagation of the beam is time independent.

Temporal coherence is also called "longitudinal coherence".

Discussion :—

A one dimensional plane progressive wave propagating along +x direction can be represented as.

$$\Psi(x,t) = Ae^{i(kx-\omega t)}.$$

If $A = |A|e^{i\phi}$ then $\Psi(x,t) = |A|e^{i(kx-\omega t+\phi)}$

$$= |A|\left[\cos(kx-\omega t+\phi) + i\sin(kx-\omega t+\phi)\right]$$

Actual displacement is given by the real part of ψ, is electric field E

$$E = |A|\cos(kx-\omega t+\phi).$$

According to above equation the displacement remain sinusoidal for all time $(-\infty < t < +\infty)$. But in actual practice no source is either a point source or monochromatic strictly. So no light is

a perfect sine wave extending over infinite space or time.

The average time interval for which the electric field vector remains sinusoidal and therefore a definite phase relationship exists, is called temporal coherence.

A beam of light travelling along +x direction. P and Q are two points on the line. Suppose the phases of the waves crossing the points P and Q at any instant t_1 is ϕ_1 and ϕ_2 and same at instant t_2 is ϕ_1' and ϕ_2'. Then the beam is said to be coherent if $\phi_2 - \phi_1 = \phi_2' - \phi_1'$.

Coherence time :- The time interval over which the phase difference remains constant is known as coherence time (τ_c)

Coherence length :- The distance over which the phase difference remains constant i.e. over which the field is sinusoidal is called coherence length (L_c)

Relation between coherence length and coherence time.

Coherence length = Coherence time × Velocity of light.

i.e, $L_c = \tau_c \times c$

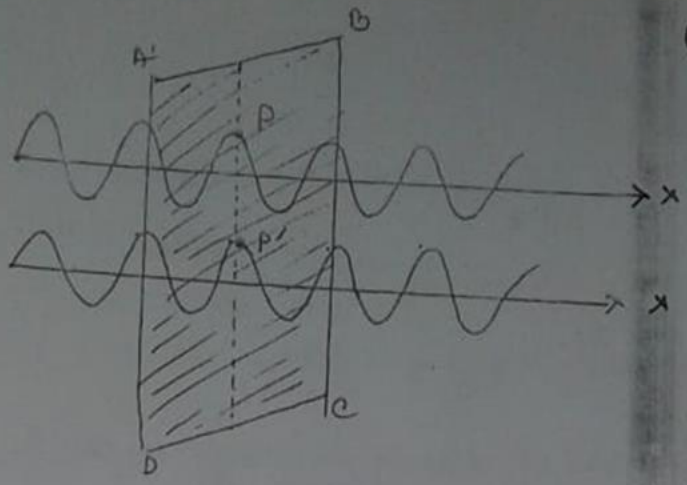
Question : Define and discuss "Spatial coherence".

Spatial coherence :-

A beam of light is said to posses spatial coherence if the phase difference of the waves crossing the two points lying on the a plane perpendicular to the direction of propagation of the beam is time independent.

(2)

Discussion :-



A beam of light travelling along +x direction is shown in fig. ABCD is a plane perpendicular to the direction of beam. P and P' are two points on the plane. Suppose the waves crossing P and P' at any time t_1 have the same phase ϕ_1 and when measured at a time t_2 again have the same phase ϕ_2, then the beam is called spatial coherent.

Define "Doppler width".

Doppler width :- The line width is caused due to uncertainty in the frequency of the wave train emitted when the orbital electron jumps from higher orbit to a lower orbit. This uncertainty in frequency is due to Doppler effect because of the motion of the electron. This line width is called Doppler width.

Relation between coherence length and line width.

If λ_1 and λ_2 are two very close wavelengths are completely out of step in Michelson interferometer when the path difference between them equals to coherent length L_c.

i.e bright ring due to λ_1 coincides with a dark ring of λ_2.

$\therefore L_c = n\lambda_1 = (n + \frac{1}{2})\lambda_2$. Where $n = L_c/\lambda_1$

$\therefore L_c = \left(\frac{L_c}{\lambda_1} + \frac{1}{2}\right)\lambda_2$

$$\therefore \frac{L_c}{\lambda_2} - \frac{L_c}{\lambda_1} = 1$$

$$\text{or, } L_c \cdot \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} = 1$$

$$\text{or, } L_c = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Instead of discrete wavelength λ_1 and λ_2, the continuous distribution of wavelength.

$$L_c = \frac{\lambda^2}{d\lambda}$$

$$* \quad \therefore \boxed{d\lambda = \frac{\lambda^2}{L_c}}$$

Frequency spread :-

$$\nu = \frac{c}{\lambda}$$

$$\therefore |d\nu| = \frac{c}{\lambda^2} |d\lambda|$$

$$= \frac{c}{\lambda^2} \cdot \frac{\lambda^2}{L_c}$$

$$= \frac{c}{L_c}$$

$$* \quad \therefore \boxed{d\nu = \frac{c}{L_c}}$$

Again $L_c = \tau_c \times c \quad \therefore d\nu = \frac{c}{\tau_c \times c} = \frac{1}{\tau_c}$

$$* \quad \therefore \boxed{d\nu = \frac{1}{\tau_c}}$$

Note :- The quantity $\frac{d\nu}{\nu}$ represents the monochromaticity or spectral purity of the source.