Semester-VI

B.Sc (Honours) in Physics

DSE 4: **Experimental Techniques**

Lecture

on

Measurements

By

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Lecture- I

Specially: Accuracy and precision and Significant figures

Syllabus

- Measurements
  - Accuracy and precision and Significant figures.
  - Error and uncertainty analysis.
  - Types of errors:
    - Gross error,
    - Systematic error,
    - Random error.
  - Statistical analysis of data
    - Arithmetic mean,
    - Deviation from mean,
    - Average deviation,
    - Standard deviation,
    - Chi-square and
  - Curve fitting.
  - Guassian distribution.

Accuracy and precision and Significant figures

Measurement

Measurement, the process of associating numbers with physical quantities and phenomena. Measurement is fundamental to the sciences; to engineering, construction, and other technical fields; and to almost all everyday activities. For that reason the elements, conditions, limitations, and theoretical foundations of measurement have been much studied.

Measurements may be made by unaided human senses, in which case they are often called estimates, or, more commonly, by the use of instruments, which may range in complexity from simple rules for measuring lengths to highly sophisticated systems designed to detect and measure quantities entirely beyond the capabilities of the senses, such as radio waves from a distant star or the magnetic moment of a subatomic particle.

Measurement begins with a definition of the quantity that is to be measured, and it always involves a comparison with some known quantity of the same kind. If the object or quantity to be measured is not accessible for direct comparison, it is converted or "transduced" into an analogous measurement signal. Since measurement always involves some interaction between the object and the observer or observing instrument, there is always an exchange of energy, which, although in everyday applications is negligible, can become considerable in some types of measurement and thereby limit accuracy.

Accuracy, Precision, and Significant Figures

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

Figure 1: A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The "known masses" are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)

The **precision of a measurement** system refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

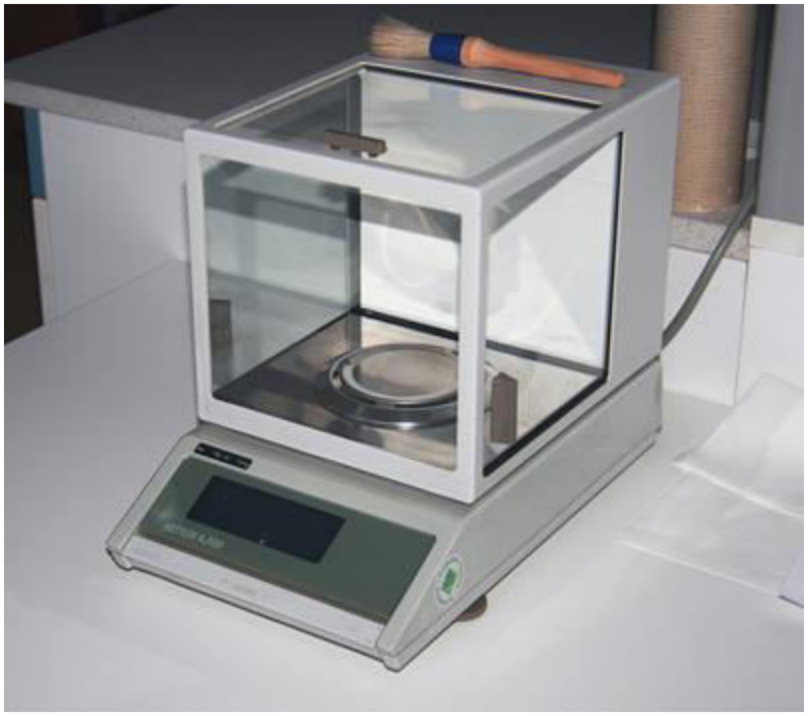

Figure 2: Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In **Figure 3**, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in **Figure 4**, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

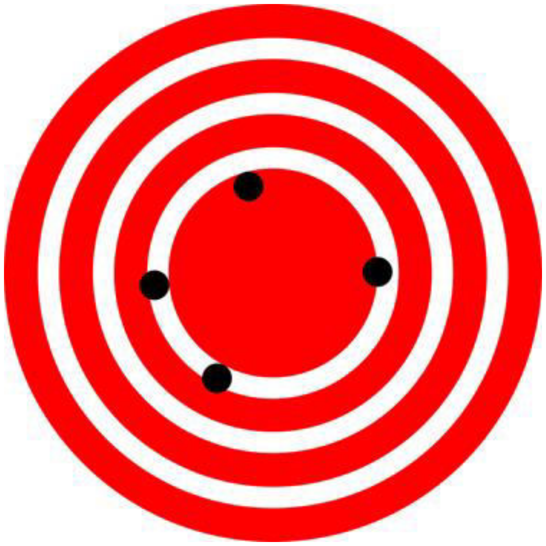

Figure 3: *A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy.*

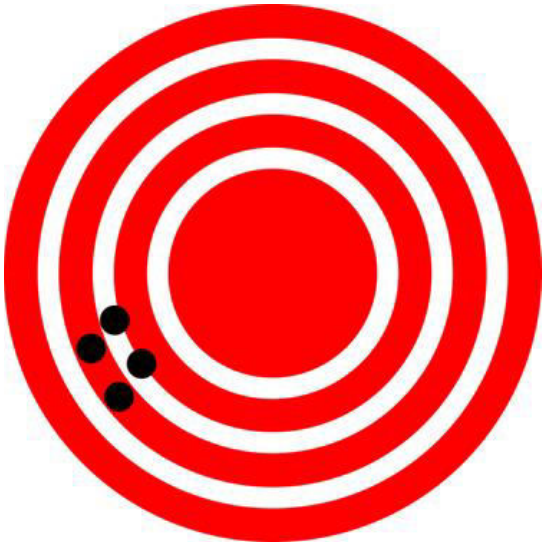

Figure 4. In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7cm. You could not express this value as 36.71cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6cm and 36.7cm, and he or she must estimate the value of the last digit. Using the method of significant figures, the rule is that *the last digit written down in a measurement* is the first digit with some uncertainty. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using A=πr2. Let us see how many significant figures the area has if the radius has only two—say, r=1.2m. Then,

$$A=\pi r2=(3.1415927...)\times(1.2m)2=4.5238934m^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A=4.5m^2$$

even though \(π\) is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement*. Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$7.56\,kg-6.052\,kg+13.7\,kg\,15.208\,kg=15.2\,kg.$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c=2\pi r,$ it does not affect the number of significant figures in a calculation.

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

Accuracy, Precision, and Uncertainty

The degree of **accuracy** and **precision** of a measuring system are related to the uncertainty in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as 11 in.$\pm$ 0.2.

The factors contributing to uncertainty in a measurement include:

- **Limitations of the measuring device,**
- **The skill of the person making the measurement,**
- **Irregularities in the object being measured,**

Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the percent uncertainty (%uncertainty) is defined to be

$$\%unc = (\delta A/A)\times 100\%$$

Example : Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

Week 1 weight: 4.8 lb

Week 2 weight: 5.3 lb

Week 3 weight: 4.9 lb

Week 4 weight: 5.4 lb

You determine that the weight of the 5-lb bag has an uncertainty of ±0.4lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, , is 5 lb. The uncertainty in this value, , is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight: $\% \text{ unc} = (\delta A / A)\times 100$

Solution

Plug the known values into the equation:

$\% \text{ unc} = \% \text{ unc} = (0.4\text{lb}/5\text{lb})\times 100 = 8$

Discussion

We can conclude that the weight of the apple bag is 5lb ± 8. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

What Is the Difference Between Accuracy and Precision?

Accuracy and precision are two important factors to consider when taking data measurements. Both accuracy and precision reflect how close a measurement is to an actual value, but accuracy reflects how close a measurement is to a known or accepted value, while precision reflects how reproducible measurements are, even if they are far from the accepted value.

Key Takeaways: Accuracy Versus Precision

- Accuracy is how close a value is to its true value. An example is how close an arrow gets to the bull's-eye center.
- Precision is how repeatable a measurement is. An example is how close a second arrow is to the first one (regardless of whether either is near the mark).
- Percent error is used to assess whether a measurement is sufficiently accurate and precise.

You can think of accuracy and precision in terms of hitting a bull's-eye. Accurately hitting the target means you are close to the center of the target, even if all the marks are on different sides of the center. Precisely hitting a target means all the hits are closely spaced, even if they are very far from the center of the target. Measurements that are both precise and accurate are repeatable and very near true values.

Accuracy, Precision, and Calibration

Do you think it's better to use an instrument that records accurate measurements or one that records precise measurements? If you weigh yourself on a scale three times and each time the number is different, yet it's close to your true weight, the scale is accurate. Yet it might be better to use a scale that is precise, even if it is not accurate. In this case, all the measurements would be very close to each other and "off" from the true value by about the same amount. This is a common issue with scales, which often have a "tare" button to zero them.

While scales and balances might allow you to tare or make an adjustment to make measurements both accurate and precise, many instruments require **calibration**. A good example is a thermometer. Thermometers often read more reliably within a certain range and give increasingly inaccurate (but not necessarily imprecise) values outside that range. To calibrate an instrument, record how far off its measurements are from known or true values. Keep a record of the calibration to ensure proper readings. Many pieces of equipment require **periodic calibration to ensure accurate and precise readings**.

Error and uncertainty analysis

Has been discussed in DSE-4-(Measurement) Lec-II slide