across the capacitor of I_1 and I_2.

The solution of above differential equation can now be obtained at the output of I_2 with the keys K_1 and K_2 opened and K_3 closed simultaneously by means of a relay at $t=0$.

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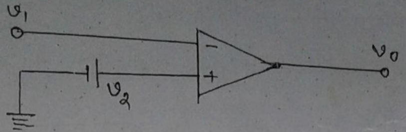
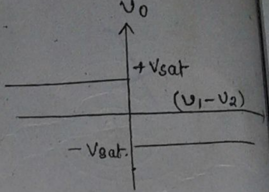
(11) * Comparator :





The comparator is a circuit which can compare two voltages.

Explaination: As long as $v_1 < v_2$ the output voltage v_0 goes to maximum positive saturation voltage $(+V_{sat})$.

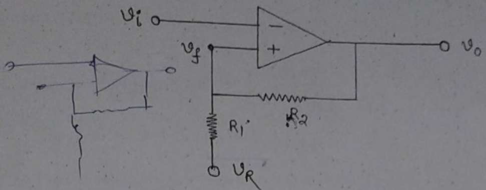
As v_1 crosses v_2 towards the region $v_1 > v_2$, the OP-AMP output switches to the negative saturation value $(-V_{sat})$.

Thus by looking at the output voltage, we can ~~ista~~ instantly identify whether ~~the~~ $v_1 > v_2$ or $v_1 < v_2$.

(12) Schmitt trigger : <Regenerative comparator>

A comparator which uses positive or regenerative feedback and exhibits the phenomena of hysteresis is called a regenerative comparator ; more commonly a Schmitt trigger.

Circuit —



A schmitt trigger circuit using an OP AMP is shown in figure.

The input voltage v_i is applied to the inverting terminal and the feedback voltage is applied to the non-inverting terminal.

i) When $v_i < v_f$, the output v_0 is at positive saturation level ($+V_{sat}$)

Then using superposition principle we get -

$$v_f = \frac{R_2}{R_1+R_2} v_R + \frac{R_1}{R_1+R_2} \cdot V_{sat} = V_1 \text{ (say)}.$$

If now v_i is now increased then output remains constant at V_{sat}, and $v_f = V_1$ (constant) until $v_i = V_1$. At this critical (Threshold) or triggering voltage, the output regeneratively switches to "$-V_{sat}$".

ii) When $v_i > v_f$, the output v_0 is at negative saturation level ($-V_{sat}$).

Using superposition principle, we get,

$$v_f = \frac{R_2}{R_1+R_2} v_R - \frac{R_1}{R_1+R_2} \cdot V_{sat} = V_2 \text{ (say)}.$$

If now v_i is decreased the output remains at "$-V_{sat}$" until and $v_f = V_2$ until $v_i = V_2$. At this triggering voltage, the output regeneratively switches to "$+V_{sat}$".

Problems

Problem-1: Suppose a sinusoidal signal $v_i = 10 \sin 2000\pi t$ mV applied to the input of OPAMP ~~inverter~~ integrator with $R = 10\,M\Omega$ and $C = 10\,\mu F$. Find the output voltage.

Solution: Output of OPAMP ~~inverter~~ integrator $v_0 = -\frac{1}{CR}\int_0^t v_i\,dt$.

$$\therefore v_0 = -\frac{1}{10\,\mu F \times 10\,M\Omega}\cdot\int_0^t 10 \sin 2000\pi t \cdot dt \text{ mV}$$

$$= -\frac{0.10}{2000\pi}\left[-\cos 2000\pi t\right]_0^t$$

$$= \frac{1}{20000\pi}\left[\sin 2000\pi t - 1\right] \text{ mV}$$

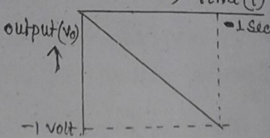
Problem-2: If the input to the OP-AMP integrator is a d.c. of 1 find the nature of output voltage. Assume $R = 1\,M\Omega$ and

Solution: Output of OP-AMP integrator,

$$v_0 = -\frac{1}{RC}\int_0^t v_i\,dt$$

$$= -\frac{1}{10^6 \times 10^{-6}}\int_0^t 1 \cdot dt$$

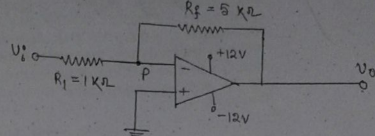
$$= -t \text{ volt}$$



Problem - 3:
V.U - 2006

Solution:

For the inverting OP-AMP circuit given below, find out the output voltage, input resistance and input current, if the input voltage is 1.5 V. d.c.



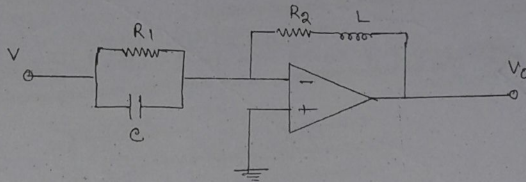
Output voltage $v_0 = -\frac{R_f}{R_1} v_i = -\frac{5}{1} \times 1.5$ volt $= -7.5$ volt.

Input resistance $R_i = R_1 = 1\ K\Omega$

Input current $i = \frac{v_i - v_p}{R_1} = \frac{(1.5 - 0)\text{ volt}}{1\ K\Omega} = 1.5$ mA.

Problem - 4: For the circuit shown in figure, calculate the output voltage
V.U - 2009

(6)



$$Z_1 = \cancel{R_1 + j\omega C} = R_1 \parallel \frac{1}{j\omega C} = \frac{R_1 + j\omega C}{j R_1 \cdot \omega C} \neq \frac{j \cancel{R_1} - \cancel{\omega C}}{R_1 \omega C}$$

$$Z_f = R_2 + j\omega L$$

Output of inverting amplifier, $V_0 = -\frac{Z_f}{Z_1} V$

$$= \frac{R_2 + j\omega L}{R_1 + j\omega C} \times j R_1 \omega C \times V$$

$$= j\left(\frac{R_2 + j\omega L}{R_1 + j\omega C}\right) R_1 \omega C V \cdot \text{volt}$$

$$\frac{R_1 + \frac{1}{j\omega C}}{\cancel{R_1 j\omega C}} \qquad \frac{\cancel{R_1 j\omega C + 1}}{R_1}$$

$$\cancel{j\omega C} + \frac{1}{R_1}$$

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2. The noninverting amplifier circuit of Fig. 14.7 has $R_f = 5\ \text{k}\Omega$ and $R_1 = 2\ \text{k}\Omega$. What is the voltage gain?

Ans. The voltage gain is

$$A_v = 1 + \frac{R_f}{R_1} = 1 + \frac{5}{2} = 3.5$$

3. When a voltage $v_1 = +40\ \mu\text{V}$ is applied to the noninverting input terminal and a voltage $v_2 = -40\ \mu\text{V}$ is applied to the inverting input terminal of an OP AMP, an output voltage $v_0 = 100\ \text{mV}$ is obtained. But, when $v_1 = v_2 = +40\ \mu\text{V}$, one obtains $v_0 = 0.4\ \text{mV}$. Calculate the voltage gains for the difference and the common-mode signals, and the common-mode rejection ratio.

Ans. In the first case, the difference signal is $v_d = v_1 - v_2 = 40 - (-40) = 80\ \mu\text{V}$ and the common-mode signal is $v_c = (v_1 + v_2)/2 = (40\text{-}40)/2 = 0$. The output voltage is

$$v_0 = A_d v_d + A_c\, v_c, \qquad \text{(i)}$$

Where A_d and A_c are the voltage gains for the difference signal and the common-mode signal, respectively, Substituting the values of v_0, v_d and v_c for the first case in (i), we obtain

$$100 \times 10^3 = A_d \times 80 + A_c \times 0.$$

or,

$$A_d = \frac{10^5}{80} = 1250$$

In the second case, $v_d = v_1 - v_2 = 40\text{-}40 = 0$ and $v_c = (v_1 + v_2)/2 = (40 + 40)/2 = 40\ \mu\text{V}$. Putting the values in (i) gives

$$0.4 \times 10^3 = A_d \times 0 + A_C \times 40$$

or,

$$A_c = \frac{0.4 \times 10^3}{40} = 10$$

The common-mode rejection ratio is

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right| = \frac{1250}{10} = 125.$$

4. Find the output voltage v_0 of the three-input summing amplifier circuit of Fig. 14.22.

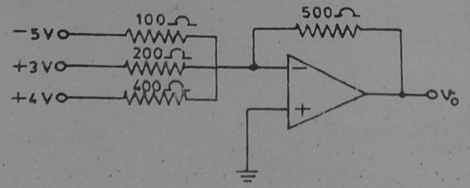

Fig 14.22

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Ans. Equation (14.17) gives for the output voltage

$$v_0 = -\left(\frac{500}{100}\times -5 + \frac{500}{200}\times 3 + \frac{500}{400}\times 4\right)$$
$$= -(-25 + 7.5 + 5) = 12.5 \text{ V}$$

5. Compute the voltage gain for the amplifier shown in Fig. 14.23. Find the output voltage v_{out} if the input voltage is $v_{in} = 0.5 \sin 100\,\pi t$ volt. (C.U. 1996).

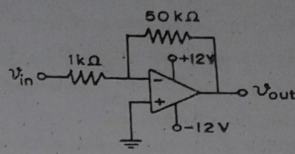

Fig. 14.23

Ans. The voltage gain of the given inverting amplifier is

$$A = -\frac{50\ \text{k}\Omega}{1\ \text{k}\Omega} = -50\,.$$

If the operation were entirely linear, the output voltage would have been

$$v_{out} = A\ v_{in} = -50 \times 0.5 \sin 100\ \pi\ t = -25 \sin 100\ \pi t \text{ V}.$$

But since the supply voltage is $\pm$ 12V, the OP AMP is saturated when $|v_{out}|$ attains 12 V. Let at time $t = t_0$, $v_{out} = -12$V. Then

$$-12 = -25 \sin 100\ \pi t_0$$

or,
$$t_0 = \frac{1}{100\,\pi}\sin^{-1}\left(\frac{12}{25}\right) = 1.59 \times 10^{-3} \text{ s}\,.$$

Thus over the entire cycle, we have

$$\begin{aligned} v_{out} &= -25 \sin 100\ \pi t \text{ V when } 0 \le t \le t_0 \\ &= -12 \text{ V when } t_0 \le t \le (0.01 - t_0) \\ &= -25 \sin 100\ \pi t \text{ V} \quad \text{when } 0.01 - t_0 \le t \le 0.01 + t_0 \\ &= +12 \text{ V} \quad \text{when } 0.01 + t_0 \le t \le 0.02 - t_0 \\ &= -25 \sin 100\ \pi t \text{ V} \quad \text{when } 0.02 - t_0 \le t \le 0.02 \text{ s.} \end{aligned}$$

The variation of v_{out} with t over a full cycle ($0 \le t \le 0.02$s) is shown in Fig. 14.24.

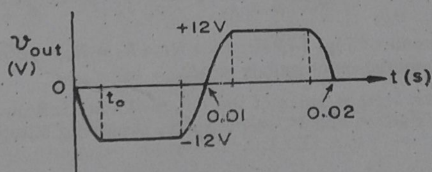

Fig. 14.24

6. A ramp voltage of 1.5 V (as shown in Fig. 14.25) per millisecond is applied to an OP AMP differentiator having $R = 2\ \text{k}\Omega$ and $C = 0.01\ \mu\text{F}$. Find the output voltage and its waveform. (C.U. 1999)

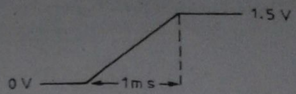

Fig. 14.25

Ans. The output voltage is $v_0 = -RC\dfrac{dv_i}{dt}$.

Here v_i is as shown in Fig. 14.25. For $0 < t < 1$ ms, $\dfrac{dv_i}{dt} = \dfrac{1.5\text{V}}{1\text{ms}}$; otherwise $\dfrac{dv_i}{dt} = 0$..

Also, $RC = 2 \times 0.01$ ms.

Hence $v_0 = -0.02 \times 1.5 = -0.03$ V $= -30$ mV for $0 < t < 1$ ms. Otherwise, $v_0 = 0$.

The waveform of v_0 is shown in Fig. 14.26.

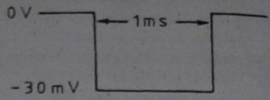

Fig. 14.26

7. In the circuit of Fig. 14.26A, express v_0 in terms of v_1 and v_2.

Ans. All the voltages are measured with respect to ground. The voltage at the point b is

$$v = \frac{R_2}{R_1 + R_2} v_2.$$

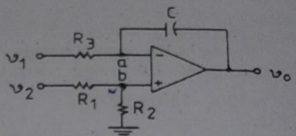

Fig. 14.26A

The voltage at the point a is also v owing to the infinite voltage gain of the OP AMP. The current through R_3 is $i = (v_1 - v)/R_3$. This current charges the capacitance C, and so the capacitor voltage is

$$v - v_0 = \frac{1}{C}\int_0^t i\,dt$$

or,
$$\frac{R_2 v_2}{R_1 + R_2} - v_0 = \frac{1}{CR_3}\int_0^t \left(v_1 - \frac{R_2 v_2}{R_1 + R_2}\right) dt$$

or,
$$v_0 = \frac{R_2 v_2}{R_1 + R_2} - \frac{1}{CR_3}\int_0^t \left(v_1 - \frac{R_2 v_2}{R_1 + R_2}\right) dt.$$

8. For the circuit of Fig. 14.26B, $R_1 = R_3 = R_4 = R_5 = 10\ \text{k}\Omega$, and $R_2 = 100\ \Omega$. Find the differential mode gain $A = v_0/(v_1 - v_2)$.

Ans. Since the voltage gains of the OP AMPs are infinite, the voltages of the points X and Y are V_1 and V_2, respectively. Applying Kirchhoff's current law at X, we obtain

$$\frac{V_1}{R_1} + \frac{V_1 - V}{R_3} + \frac{V_1 - V_2}{R_2} = 0$$

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Problem 4. *If the circuit of Fig. 10.10-1 has $R_1 = 10\,\text{k}\Omega$ and $R_2 = 50\,\text{k}\Omega$ what would be the output voltage for an input (a) $v_s = 1$ V and (b) $v_s = 3$ V. Assume OP AMP supply voltages to be ± 12 V*

Solution :

(a) The output voltage

$$v_0 = \left(1 + \frac{R_2}{R_1}\right) v_s = \left(1 + \frac{50}{10}\right).1\,\text{V}$$
$$= 6\,\text{V}$$

(b) If we put $v_s = 3$ V in the above relation v_0 comes out to be 18 V. But actually the OP AMP, in this case, saturates and v_0 becomes very nearly equal to $+12$ V (supply voltage).

Problem 5. *Calculate the output voltage of the OP AMP summing amplifier of Fig. 10.10-3 with three inputs $v_1 = 1$ V, $v_2 = -2$ V and $v_3 = +3$ V. Assume $R_1 = R_2 = R_3 = 500\,\text{k}\Omega$ and $R_f = 1\,\text{M}\Omega$.*

Solution :

$$v_0 = -\frac{R_f}{R_1}(v_1 + v_2 + v_3)$$
$$= -\frac{1\,\text{M}\Omega}{500\,\text{k}\Omega}(1 - 2 + 3)\,\text{V}$$
$$= -2 \times 2\,\text{V}.$$
$$= -4\,\text{V}$$

Problem 6. *Suppose a sinusoidal signal $v_s = 10 \sin 2000\pi t$ mV is applied to the input of the OP AMP integrator of Fig. 10.10-5 with $R = 1\,\text{M}\Omega$ and $C = 1\,\mu\text{F}$. Find the output voltage.*

Solution :

$$v_0 = -\frac{1}{CR}\int_0^t v_s dt = \frac{1}{(1\,\mu\text{F})(1\,\text{M}\Omega)}\int_0^t 10 \sin 2000\pi t dt\,\text{mV}$$
$$= -0.10\left[\frac{-\cos 2000\pi t}{2000\pi}\right]_0^t \text{mV}$$
$$= \frac{1}{200\pi}[\cos 2000\pi t - 1]\,\text{mV}$$

Problem 7. *If the input to the OP AMP integrator of Fig. 10.10-5 is a d.c. of 1 V, find the nature of output voltage. Assume $R = 1\,\text{M}\Omega$ and $C = 1\,\mu\text{F}$.*

Solution :

$$v_0 = -\frac{1}{CR}\int_0^t v_s dt = -\frac{1}{1\,\mu\text{F}}1\,\text{M}\Omega\int_0^t 1.dt\ \text{volt}$$
$$= -t\ \text{volt}$$

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Thus the output is a linearly decreasing ramp voltage. The output ramp reaches −1 V in 1 second (Fig. 10.P-7).

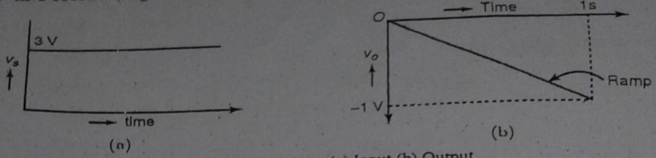

Fig. 10.P-7: Integrator (a) Input (b) Output

Problem 8. *A ramp voltage of* 1.5 V *per millisecond (as shown in Fig.* 10.P-8(a)) *is applied to an* OP AMP *differentiator with* $F = 2\,\text{k}\Omega$ *and* $C = 0.01\,\mu\text{F}$. *Find the output voltage and its waveform.* (C.U. 1999)

Solution : From Fig. 10.P-8(a)

$$\frac{dv_s}{dt} = 1.5\ \text{V/ms within the 1 ms range}$$

$$= 0\ \text{outside the 1 ms range}$$

Now output

$$v_0 = -CR\frac{dv_s}{dt}$$

$$= -(0.01\,\mu\text{F})(2\,\text{k}\Omega) \times 1.5\ \text{V/ms}$$

$$= -0.0 \times 10^{-6} \times 2 \times 10^3 \times 1.5 \times 10^3\ \text{V}$$

$$= -0.03\ \text{V}$$

$$= -30\ \text{mV within the 1 ms range}$$

$v_0 = 0$ outside the 1 ms range

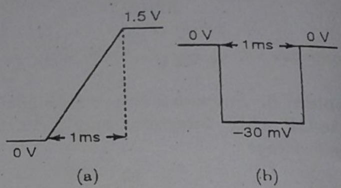

Fig. 10.P-8: Integrator (a) Input (b) Output

Therefore, the output waveform will be as shown in Fig. 10.P-8(b).

Problem 9. *What is the output voltage in the circuit of Fig.* 10.P-9? *Explain the operation of circuit.* (C.U. 2000)

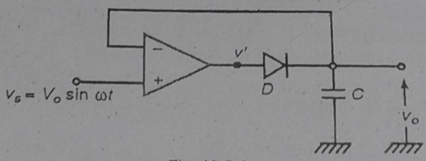

Fig. 10.P-9

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Solution : If $v_s > v_0$ the OP AMP output v' is positive. The diode D then conducts and capacitor C is charged to the value of the input because the gain of the circuit is unity. In this way the capacitor is charged to the peak value (V_0) of the input. When v_s goes below the capacitor voltage OP AMP output v' becomes negative. The diode then becomes reverse-biased and stops conducting. However the capacitor voltage remains at the peak value (V_0) of the input. Thus the circuit acts as a *peak detector*.

Problem 10. *If a sinusoidal voltage* $v_s = 10 \sin 2000\pi t$ mV *is applied to the input of the* OP AMP *differentiator of Fig.* 10.10-6 *with* $C = 1\,\mu F$ *and* $R = 100\,k\Omega$, *what would be the output voltage?*

Solution :
$$v_0 = -CR\frac{dv_s}{dt} = -10^{-6} \times 10^5 \times \frac{d}{dt}(10 \sin 2000\pi t)\text{ mV}$$
$$= -2000\pi \cos 2000\pi t\text{ mV}$$
$$= -2\pi \cos 2000\pi t\text{ V}$$

Problem 11. *Consider the* OP AMP *amplifier of Fig.* 10.9-1 *with* $R_1 = 1\,k\Omega, R_2 = 50\,k\Omega$ *and* OP AMP *supply voltages* ± 12 V. *Compute the gain and find the output if input is* $V_s = 0.5 \sin 100\pi t$ *volt.* (C.U. 1996)

Solution : Gain of the amplifier is

$$-\frac{R_2}{R_1} = -50$$

If the OP AMP were within linear region over the whole range of input, then the output would have been

$$v_0 = -50v_s = -25 \sin 100\pi t \text{ volt}$$

Since the supply voltages are ± 12 V, the output saturates at ± 12 V. So the output will be sinusoidal voltage $-25 \sin 100\pi t$ volt with sharp clippings at ± 12 V as shown in Fig. 10.P-11.

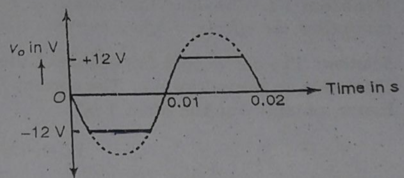

Fig. 10.P-11

Problem 12. *Draw a circuit using one or more* OP AMP *whose output* v_0 *is given as* $v_0 = 4v_1 + 6v_2$, *where* v_1 *and* v_2 *are two input signals.* (B.U. 1999)

Solution : In Fig. 10.P-12 $v' = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right) = -(4v_1 + 6v_2)$

Now the inverter output $v_0 = -v' = 4v_1 + 6v_2$

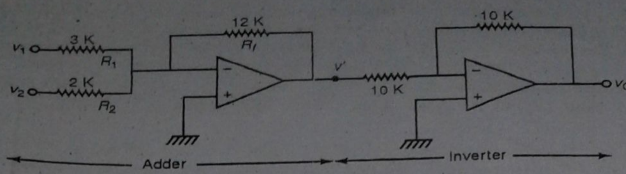

Fig. 10.P-12

Problem 13. *What does the circuit of Fig. 10.P-13 do ? Explain.*

Solution : The input v_1 drives an inverter of gain unity. So the output of first stage is $-v_1$. The second stage is summing amplifier with gain unity for each input. Thus $v_0 = v_1 - v_2$ and the circuit acts as a *subtractor.*

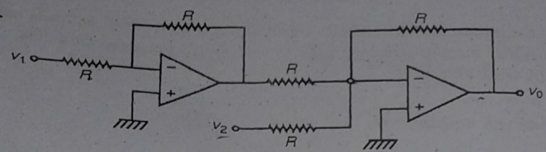

Fig. 10.P-13

Problem 14. *Consider the circuit of Fig. 10.P-14. Find the value of R_3 which minimises the effects of offset current. Assume output impedance to be zero.*

Solution : The d.c. output impedance of the OP AMP is low and the output terminal is effectively at ground for d.c. The current I_1 passes the resistors R_1 and R_2 in parallel. Hence, potential at the point 1 is

$$V_1 = -I_1 \cdot \frac{R_1 R_2}{R_1 + R_2}$$

The potential at the point 2 is

$$V_2 = -I_2 \cdot R_3$$

$\therefore$ The output voltage of the amplifier is given by

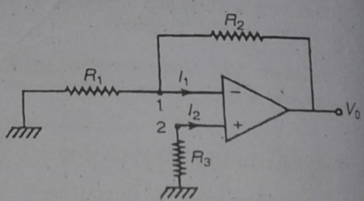

Fig. 10.P-14

$$V_0 = A(V_2 - V_1)$$

$$= A\left(I_1 \frac{R_1 R_2}{R_1 + R_2} - I_2 R_3\right)$$

Thus $V_0 = 0$ if we choose $I_1 \cdot \frac{R_1 R_2}{R_1 + R_2} = I_2 R_3$

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Therefore $R_3 = \frac{R_1 R_2}{R_1+R_2}$ is the desired value of R_3 if $I_1 \approx I_2$. Note that R_3 has no effect on the virtual ground approximation because no a.c. singal current flows through it.

✓**Problem 15.** *Consider the differential-input amplifier of Fig. 10.P-15. Assuming infinite input resistance, zero output resistance and finite differential gain* $A_V = V_0/(V_1 - V_2)$ *show that closed loop gain is given by* $A_{Vf} = \frac{A_V}{1+\frac{A_V}{n+1}}$. *Discuss the case* $A_V \to \infty$. (cf. C.U. 2008)

Solution:

$$V_2 = \frac{V_0 . R}{nR + R} = \frac{V_0}{n+1}$$

$$V_1 = V_s$$

$$\therefore \quad V_0 = A_V(V_1 - V_2)$$

$$= A_V\left(V_s - \frac{V_0}{n+1}\right)$$

$$\text{or} \quad A_{Vf} = \frac{V_0}{V_s} = \frac{A_V}{1+\frac{A_V}{n+1}}.$$

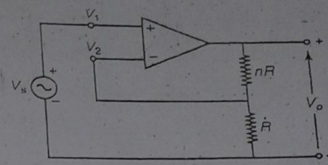


Fig. 10.P-15

In the limit $A_V \to \infty$, $A_{Vf} \to n+1$.

✓**Problem 16.** *In a differential amplifier with two inputs the output is* 2.01 mV *when the inputs are* $110\,\mu$V *and* $90\,\mu$V *but the output is* 2 mV *when inputs are* $10\,\mu V$ *and* $-10\,\mu$V. *Find the* CMRR *of the amplifier.* (B.U. 1999)

Solution: In the second case $v_d = 20\,\mu$V and $v_c = 0$

$$v_0 = A_d v_d + A_c v_c \quad \text{or} \quad 2\,\text{mV} = A_d \times 20\,\mu\text{V}$$

$$\therefore \quad A_d = 100$$

In the first case,

$$v_d = 20\,\mu\text{ V}, \quad v_c = \frac{v_1 + v_2}{2} = 100\,\mu\text{ V}$$

$$\therefore \quad v_0 = A_d v_d + A_c v_c$$

$$\text{or,} \quad 2.01\,\text{mV} = 100 \times 20\,\mu\text{V} + A_c \times 100\,\mu\text{V}$$

$$\text{or,} \quad A_c = \frac{1}{10}$$

$$\therefore \quad \text{CMRR} = \left|\frac{A_d}{A_c}\right| = 1000$$

Problem 17. *Show that the output voltage of the circuit in Fig. 10.P-17 is given by*

$$v_0 = v_2 \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} - v_1 \cdot \frac{R_2}{R_1}$$

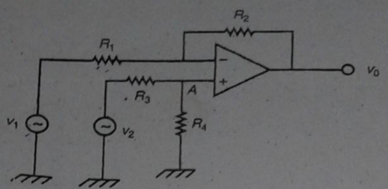


Fig. 10.P-17

(cf. B.U. 2004)

Solution: We can find v_0 by using superposition principle. The output voltage due to v_1 alone is

$$v_{01} = -\frac{R_2}{R_1} \cdot v_1$$

Voltage at the non-inverting terminal is $v_A = \frac{v_2 \cdot R_4}{R_3 + R_4}$. Therefore, the output voltage due to v_2 alone is

$$v_{02} = \left(1 + \frac{R_2}{R_1}\right) v_A = v_2 \cdot \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1}$$

Hence by the superposition principle, the resultant output voltage is

$$v_0 = v_{01} + v_{02} = v_2 \cdot \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} - v_1 \cdot \frac{R_2}{R_1}$$

Problem 18. *Find the output voltage v_0 in the circuit of Fig. 10.P-18.* (GATE 2001)

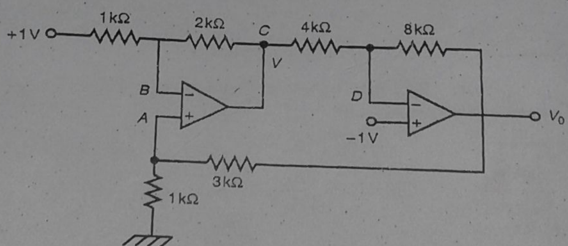
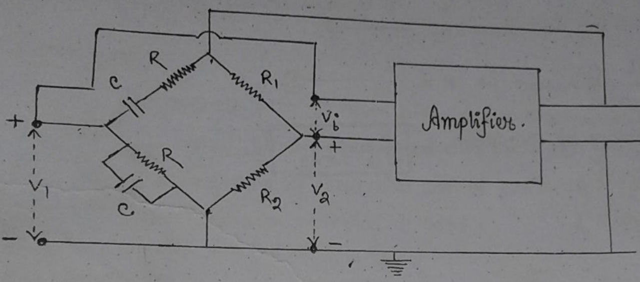


Fig. 10.P-18

Wien-Bridge Oscillator

V.U - 2001, 2003, 2007

Oscillators using tuned (LC) circuits are not suitable for generating low frequency (LF) signals because then the physical dimensions of L and C would be large and hence the device would be costly. That is why for generating low frequency signals RC oscillators are used. Wien-Bridge oscillator is one such RC oscillator which is widely used in the laboratory as an audio frequency (AF) signal generator.



The circuit diagram of a Wien-Bridge oscillator using amplifier is shown in figure. Either OPAMP or BJT or FET can be used as amplifier.

The frequency determining circuit of the amplifier is a wien-bridge circuit. The R-C combination and the resistors R_1 and R_2 form the four arms of a Wien-bridge.

The ~~output~~ Input V_i of the bridge is amplified by the amplifier. The output V_o of the amplifier becomes the bridge supply voltage.

When the bridge is exactly balanced, i.e. $V_i = 0$. But for sustained oscillation, v_i must be nonvanishing. This is done by adjusting the ratio between the resistances R_1 and R_2.

Frequency of oscillation :

When the bridge is balanced, we have

$$\frac{R_1}{R_2} = \frac{Z_1}{Z_2} = \frac{R + \frac{1}{j\omega C}}{R \parallel \frac{1}{j\omega C}} = \frac{R + \frac{1}{j\omega C}}{\frac{R/j\omega C}{R + 1/j\omega C}}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{(R + 1/j\omega C)^2}{R/j\omega C}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{R^2 - 1/\omega^2 C^2 + 2R/j\omega C}{R/j\omega C}$$

$$\Rightarrow \frac{R_1}{R_2} = 2 + j\,\frac{R^2 - 1/\omega^2 C^2}{R/\omega C} \quad \text{—— (1)}$$

Equating the real and imaginary part of above equation,

$$\frac{R_1}{R_2} = 2 \quad \text{i.e} \quad R_1 = 2R_2 \quad \text{—— (2)}$$

and

$$\frac{R^2 - 1/\omega^2 C^2}{R/\omega C} = 0 \quad \text{i.e} \quad R = 1/\omega C \quad \text{—— (3)}$$

The frequency of oscillation, $\boxed{f = \frac{1}{2\pi RC}}$ —— (4)

Amplifier gain :

When the bridge is balanced, the voltage across the two lower arm of the bridge must be equal i.e.

$$V_1 = V_2 = \frac{R_2}{R_1 + R_2} V_0 \text{ —— (5)} \quad \text{as } V_2 = \frac{R_2}{R_1 + R_2} V_0 \quad \therefore \boxed{A = \frac{V_0}{V_1} = \frac{R_1 + R_2}{R_2}}$$

Again, at balance condition, $\frac{R_2}{R_1 + R_2} = 1/3$ using equation (2)

$$\therefore \frac{V_1}{V_0} = \frac{V_2}{V_0} = \frac{1}{3} \quad \text{—— (6)}$$

oscillation to occour, $V_i \neq 0$ then the ratio $\frac{R_2}{R_1 + R_2}$ must be less.

Let $\frac{R_2}{R_1 + R_2} = \frac{1}{3} - \frac{1}{\delta}$ Where δ is a number greater than 3.

$$\frac{V_2}{V_0} = \frac{1}{3} - \frac{1}{\delta} \quad \text{—— (7)}$$

As the bridge is unbalanced by adjusting R_1/R_2, the ratio $\frac{V_1}{V_0}$ remains $1/3$.

but the ratio $\frac{V_2}{V_0}$ becomes, $\frac{V_2}{V_0} = \frac{1}{3} - \frac{1}{\delta}$

Thus feedback voltage is, $V_i = V_1 - V_2 = \left(\frac{1}{3} - \frac{1}{3} + \frac{1}{\delta}\right) V_0 = \frac{V_0}{\delta}$

Clearly, V_i and V_0 are in phase and feedback fraction $\beta = \frac{V_i}{V_0} = \frac{1}{\delta}$

Thus the condition $\beta A = 1$ is satisfied by making the gain of the amplifier $A = \delta$ where δ is greater than 3.

Thus the amplifier used in Wien bridge oscillator must have a gain greater than 3.

Advantages :

i) The overall gain is high since a two stage amplifier is used.
ii) The circuit gives a very good sine wave output.
iii) The frequency of oscillation can be easily varied.
iv) The frequency stability is good.

Disadvantages :

i) The circuit is not capable of generating very high frequency.
ii) The number of components required for a two stage amplifier is very large.

Problem :
V.U - 2005

A wien-bridge oscillator has a frequency of 1000 Hz and a capacitance of 100 pF. Find the resistance :

If the amplifier gain is 10, find the ratio of resistances in the other arms.

Solution : $f = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 1000 \times 10^{-10}}$ ohm

$= 1.59\ M\Omega$

Gain of the amplifier $A = \frac{R_1 + R_2}{R_2} \Rightarrow 10 = \frac{R_1}{R_2} + 1 \Rightarrow \boxed{\frac{R_1}{R_2} = 9 : 1}$