

Radioactivity

① Radioactivity is the phenomena of spontaneous disintegration, with emission of corpuscular or, e.m. radiations, of heavy atomic nuclei like U, Ra etc. at a constant rate, unaffected by any physical or chemical change. to which the atom may be subjected. It is a nuclear property of the active element and in all radioactive processes, a transmutation of the elements occurs and a new nucleus is formed. Radiations from diffⁿ radioactive substances were classified as α -rays and β -rays by Rutherford from a study on their penetrating power. Later, a third energetic radiation, γ -rays, was discovered by Villard.

Radioactive decay law

① One emission of α or β -rays which is usually, but not invariably, accompanied by γ -emission, the emitting parent nuclide transforms into a new daughter element; the daughter element again may be radioactive so that the process of successive disintegration continues till the original active parent nuclide gets transformed into a stable one, usually lead (Pb)

② The rate of radioactive disintegration, that is, the number of atoms that disintegrate at any instant t is directly proportional to the number N_t of the active nuclides present in the sample under study at that instant.

Decay law or decay equation:-

Let N_t be the number of active nuclides present in the sample at any instant t . Then, we have, experimentally -

$$-\frac{dN_t}{dt} \propto N_t \Rightarrow \frac{dN_t}{dt} = -\lambda N_t \quad \text{--- ①}$$

where λ , the constant of proportionality, is known as the decay constant - a characteristic const. of the element. The \ominus -ve sign indicates that N_t decreases with t .

Re-arranging eqⁿ ①, we obtain

$$\frac{dN_t}{N_t} = -\lambda dt \quad \text{--- ②}$$

Intⁿ the eqⁿ ②, we have

$$\ln N_t = -\lambda t + A \quad \text{--- ③}$$

where A is the const.

At $t=0$, $N_t = N_0$, the initial number of nuclides.

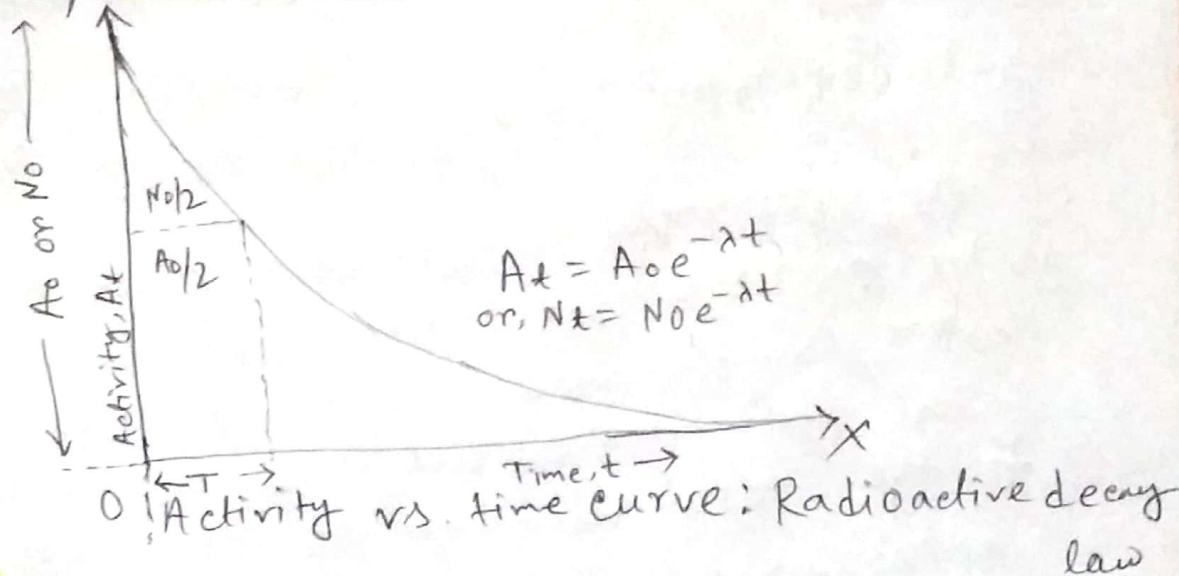
So, from ③

$$A = \ln N_0$$

From (3), therefore, we obtain finally,

$$\ln(N_t/N_0) = -\lambda t$$

or, $N_t = N_0 e^{-\lambda t}$ — (4)



Half life

The half-life of a radioactive nuclide is defined as the time T in which the original amount of radioactive nuclide is reduced by way of disintegrations to half its value.

Substituting N_t by $N_0/2$ in $N_t = N_0 e^{-\lambda t}$,

T_h is given by $\frac{N_0}{2} = N_0 e^{-\lambda T}$

$$T_h = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\text{or, } \lambda T_h = 0.693 = \text{const.}$$

Mean or Average life:

The fundamental law of radioactive decay, as already indicated, is a statistical law implying that the probability of decay of a given nuclide in a short time interval dt at time t is $|dN_t/N_t| = \lambda dt$ which is independent of the age of the nuclide.

The average or mean life $\bar{\tau}$ of a radioactive element is the average life-time of all the atoms in the given sample and is defined as the ratio of the total life-time of all the atoms or nuclei to the total number of atoms or nuclei.

$$\begin{aligned}\therefore \bar{\tau} &= \frac{\text{Total life time of all nuclei}}{\text{Total number of nuclei}} \\ &= \frac{t_1 dN_1 + t_2 dN_2 + \dots}{dN_1 + dN_2 + \dots} = \frac{\sum t dN}{\sum dN} \\ &= \frac{\int t dN}{\int dN} = \frac{\int t dN}{-N_0} \quad \text{--- (1)}\end{aligned}$$

Where dN_1 atoms have a life time t_1 , dN_2 atoms a life-time t_2 and so on.

But, we have, $dN = d(N_0 e^{-\lambda t}) = -\lambda N_0 e^{-\lambda t} dt$
 $\Rightarrow t dN = -\lambda t N_0 e^{-\lambda t} dt$

$$\therefore \bar{\tau} = \lambda \int_0^{\infty} t e^{-\lambda t} dt, \text{ substituting for } t dN \text{ in (1)}$$

$$= \lambda \left[-\frac{t}{\lambda} e^{-\lambda t} + \frac{1}{\lambda} \int_0^{\infty} e^{-\lambda t} dt \right]$$

$$= \lambda \left[-\frac{t}{\lambda} e^{-\lambda t} - \frac{1}{\lambda^2} e^{-\lambda t} \right]_0^{\infty} = \frac{1}{\lambda}$$

\therefore Mean or Average life, $\boxed{\tau = \frac{1}{\lambda} = 1.443 T_{1/2}}$ — (2)

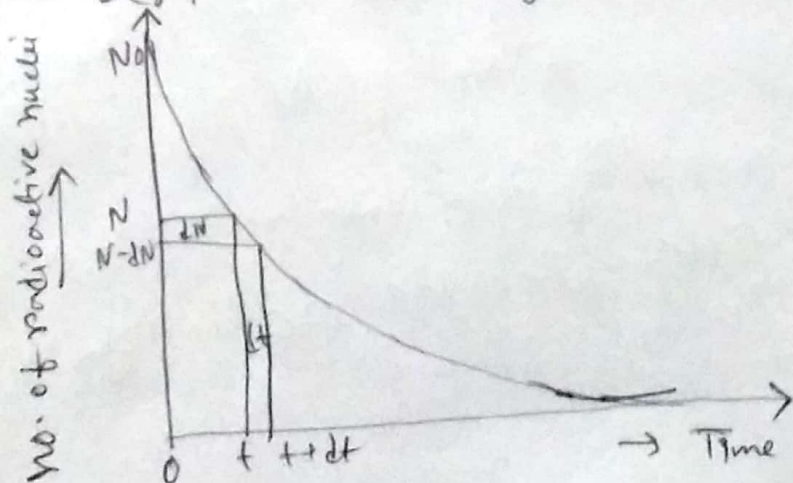
The mean or average life of a radioactive element is thus not the same as its half-life. The mean life is the reciprocal of the decay constant. That is, the decay probability per second and is greater than the half-life.

① To get the eqⁿ (1), the curve of fig below is illuminating. The curve shows that each of dN number of active nuclei has lived a life-time of t seconds, i.e. the total life span of dN nuclei is $dN \cdot t$ seconds.

② At $t = \tau$, then from the decay law

$$N_t = N_0 e^{-\lambda t} = N_0 e^{-1} = N_0/e \quad \text{--- (3)}$$

eqⁿ (3) implies that the mean life is the time for the nuclei in an isotope to decay to $1/e$ of their original number.



Activity or strength of a radio-sample

In the decay process, one is essentially interested in the number of disintegrations per second, called activity A of the sample.

$$\therefore A = \left| \frac{dN_t}{dt} \right| = -\lambda N_0 e^{-\lambda t} = \lambda N \quad \text{--- (1)}$$

Where $t=0$, $\left(\frac{dN_t}{dt}\right)_0 = -\lambda N_0$, Hence, from (1) above, we have

$$\frac{dN_t}{dt} = \left(\frac{dN_t}{dt}\right)_0 e^{-\lambda t} \quad \text{--- (2)}$$

$$\text{or, } \boxed{A_t = A_0 e^{-\lambda t}} \quad \text{--- (3)}$$

Where $A_t = dN_t/dt$ and $A_0 = \left(dN_t/dt\right)_0 = \text{original activity} = \lambda N_0$

Specific activity.

The activity or strength A_t of a radioactive sample at any instant t is defined as the number of disintegrations occurring in the sample in unit time at t , that is, -

$$\text{Activity, } A_t = \left| \frac{dN_t}{dt} \right| = \lambda N_0 e^{-\lambda t} = \lambda N_t = \frac{0.693}{T_h} N_t$$

The activity per unit mass of a sample is called its specific activity.

Units of activity

The earlier unit of radioactivity, still in wide use, is called the curie (Ci) and is defined (since 1950) as the activity of any radioactive substance that disintegrates at the rate of 3.70×10^{10} disintegrations per second which is the activity of 1g of radium 226

A thousandth part of a curie is called a milli curie (mCi). Still smaller unit is the micro-curie (μ Ci). So, by definition,

$$1 \text{ Ci} = 1 \text{ curie} = 3.7 \times 10^{10} \text{ disint/sec}$$

$$1 \text{ mCi} = 10^{-3} \text{ curie}$$

$$1 \mu\text{Ci} = 10^{-6} \text{ curie}$$

The SI unit of activity is the becquerel (Bq)

$$1 \text{ Bq} = 1 \text{ disintegration/second, so}$$

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

• We shall now find the quantity of U-238 having 1 curie of activity, for U-238

$$\lambda N U = 1 \text{ Ci} = 3.70 \times 10^{10} \text{ disint/s, and}$$

$$\lambda U = \frac{0.693}{T} = \frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 3600} = 4.88 \times 10^{-18}$$

$$\therefore N U = \frac{3.70 \times 10^{10}}{\lambda U} = \frac{3.70 \times 10^{10}}{4.88 \times 10^{-18}} = 7.58 \times 10^{27}$$

Amount of U-238, having 1 curie of activity is given by -

$$m_U = \frac{7.58 \times 10^{27} \times 238}{6.02 \times 10^{26}} = 2995 \text{ kg}$$

General Properties of α , β and γ -rays:-

α -rays:- The α -rays consist of material particles with a mass four times the protomic mass and a positive charge twice that of the proton. These particles are identified as helium nuclei ${}^4_2\text{He}$. They can ionise gases and penetrate matter.

β -rays:- The β -rays also consist of material particles but with a negligible mass and a negative charge. The specific charge of β -rays agree with that of the electrons and the β -rays are, in fact, identified as electrons ${}^0_{-1}\text{e}$. They can ionise gases, affect photographic plate and penetrate matter.

γ -rays

They are non-material and unchanged in character, and are unaffected by electric and magnetic fields. In fact, these consist of electromagnetic radiations of very small wave lengths, smaller than even the shortest x-rays. Like the α - and β -rays, γ -rays also affect photographic plates, ionise gases through which they pass and penetrate matter.