2.6 General properties of α, β and γ-rays

We enumerate below some of the important characteristics of α, β and γ-rays.

α-rays — The α-rays consist of *material* particles with a mass four times the protonic mass and a *positive charge* twice that of the proton. These particles are identified as *helium nuclei* $^4_2$He. They can ionise gases and penetrate matter. The velocities with which α-particles are ejected from a radioactive substance are very high, ranging from $0.03\,c$ to $0.07\,c$ where $c = 3 \times 10^8$ m/s, the velocity of light in free space.

β-rays — The β-rays also consist of *material* particles but with a *negligible mass* and a *negative charge*. The *specific charge* of β-rays agree with that of the electrons and the β-rays are, in fact, identified as *electrons* $^{\;0}_{-1}$e. They can ionise gases, affect photographic plate and penetrate matter. The penetrating power is about 100 times greater than that of the α-particles. Because of the low mass, the ionising power is only about 1/100th of that of the α-particles. The velocity of the emitted β-rays from radioactive sources depends on the source and is so high as to sometime approach close to the velocity of light in vacuo.

γ-rays — These are *non-material* and *uncharged* in character, and are unaffected by electric and magnetic fields. In fact, these consist of *electromagnetic radiations* of very small wavelengths, smaller than even the shortest x-rays. Like the α-and β-rays, γ-rays also affect photographic plates, ionise gases through which they pass (photoelectric effect) and penetrate matter. In so far as the penetrating power is concerned, they are 10–100 times more effective than β-rays, but the ionising power is proportionately much weaker.

2.7 Radioactive decay : Conservation laws

The radioactive disintegrations are governed by the following conservation laws:

1. **Energy conservation**—A nucleus A decays into a lighter one A' with emission of one or more particles if the rest energy of A is greater than that of $(A' + x)$, x being the emitted particles. The excess rest energy is known as the *Q-value* of the decay. The decay $A \rightarrow A' + x$ is possible if Q is positive.

2. **Conservation of linear momentum**—If the decaying nucleus A is at rest, the sum of the total momentum of the decay products is zero.

$$\vec{p}_{A'} + \vec{p}_x = 0$$

3. **Conservation of angular momentum**—The angular momentum of the particle before decay is equal to the total angular momentum of all products after decay.

4. **Conservation of charge and nuclear number**—The total electric charge and the total number of nucleons must not change in any decay process.

2.8 Radioactive series : Displacement law

The α-particles having been identified as helium nuclei ($^4_2$He), an α-emission by a parent nuclide of atomic number Z and mass number A transforms it into a daughter nuclide of atomic number $(Z-2)$ and mass umber $(A-4)$. Hence, it becomes a *new element* whose position in the periodic table is two places lower. The α-emission process may be represented as

$$^A_Z X \longrightarrow {}^{A-4}_{Z-2} Y + {}^4_2\text{He} \tag{2.8.1}$$

Parent element → daughter product + α-particle.

For examples:

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He}$$

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^4_2\text{He}$$

Similarly, β-particles having been identified as electrons, a β-emission changes the parent nuclide (Z, A) into a daughter $(Z+1, A)$. The mass number remains unaltered as the electronic mass is negligible. According to the present day knowledge, electron does *not exist* in the nucleus; the neutron in the nucleus spontaneously transforms into a proton during β-decay, keeping the A-value of the parent nuclide unchanged. The new element formed thus advances along the periodic table by one place. The β-emission process may be represented as

$$^A_Z X \longrightarrow {}^{A}_{Z+1} Y + {}^{0}_{-1}e \tag{2.8.2}$$

For examples:

$$^{234}_{90}\text{Th} \longrightarrow {}^{234}_{91}\text{Pa} + {}^{0}_{-1}\text{e}$$

$$^{210}_{83}\text{Bi} \longrightarrow {}^{210}_{84}\text{Po} + {}^{0}_{-1}\text{e}$$

The new element formed by β-emission is an *isobar* to the parent nucleus.

• The fact that an element comes down by two places in the periodic table by an α-emission and shoots up by one place by a β-emission constitutes the ***displacement law*** of Soddy and Fajan, enunciated empirically in 1913, when our present day knowledge regarding neutron-proton structure of nuclei was absent.

• Note that during a radioactive transformation, the mass number and the total charge is conserved. The examples of α-decay (2.8.1) and β-decay (2.8.2) are examples of *nuclear reactions*. Radioactivity is entirely a *nuclear phenomenon*; the radioactive radiations come out of the atomic nucleus.

Radioactive series — Now, the daughter of a parent radionuclide may itself be radioactive and decay in its turn into another radionuclide and the process may be repeated till the product is stable, that is, non-radioactive. These *successive transformations* of a radionuclide on being studied are generally found to lie in the range of atomic number $Z = 81$ to $Z = 92$ and form what is called a *radioactive series*. A radioactive series is named after the longest-lived member in it.

There are *three* naturally occurring radioactive series showing successive transformations. These are: (i) ***uranium series***, (ii) ***thorium series*** and (iii) ***actinium series***.

Uranium series — The uranium series starts with $^{238}_{92}U$ and ends at $^{206}_{82}Pb$, a stable isotope of lead. The series is also called $(4n+2)$ *series*, for the mass number of any nuclide in the series is given by $A = 4n + 2$, n being an integer. The (N, Z)-chart of the uranium series is given in Fig. 2.6.

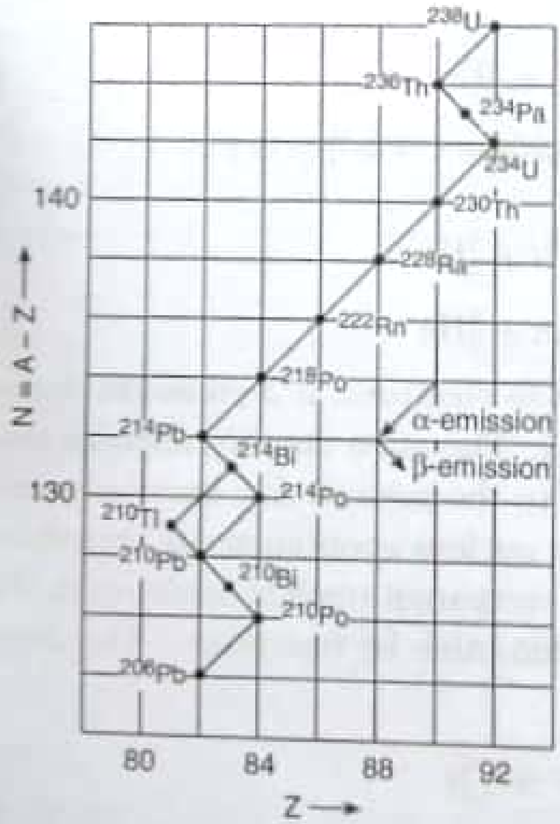

Fig. 2.6 Uranium series

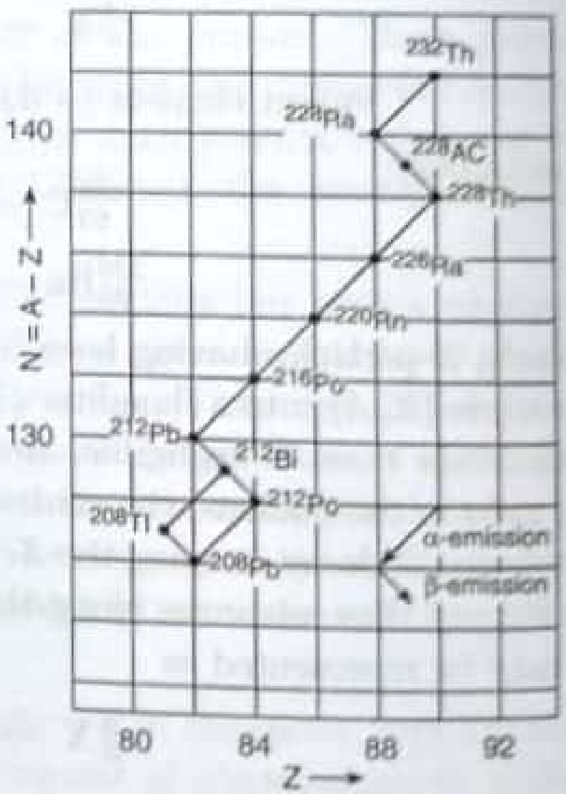

Fig. 2.7 Thorium series

Thorium series — The thorium series begins with $^{232}_{90}Th$ and ends at $^{208}_{82}Pb$, another stable isotope of lead. Since the mass number of any nuclide in the series is given by $A = 4n$, where n is an integer, it is sometimes known as $4n$-*series* and its (N, Z)-chart is given in Fig. 2.7.

Actinium series — The actinium series starts with actino-uranium, that is ${}^{235}_{92}U$ and ends at ${}^{207}_{82}Pb$, a third stable isotope of lead. The mass number of any nuclide in the series is given by $A = 4n + 3$, n being an integer. The series is therefore sometimes called $(4n+3)$ *series*. Its (N, Z)-chart is shown in Fig. 2.8.

The discovery of these three radioactive families is largely due to the pioneering and monumental work conducted by Soddy.

Table 2.1 : The four radioactive series

No.	Name of the series	Mass number A	Parent
1.	Thorium Series	$A = 4n$	${}^{232}_{90}Th$
2.	Uranium Series	$A = 4n + 2$	${}^{238}_{92}U$
3.	Actinium Series	$A = 4n + 3$	${}^{235}_{92}U$
*4.	Neptanium Series	$A = 4n + 1$	${}^{237}_{93}Np$

*This series is man-made. Other three are naturally occurring.

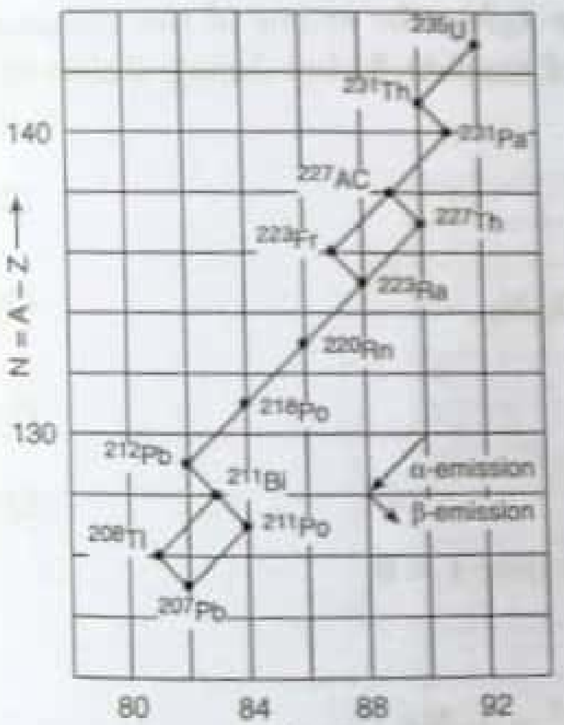

Fig. 2.8 Actinium series

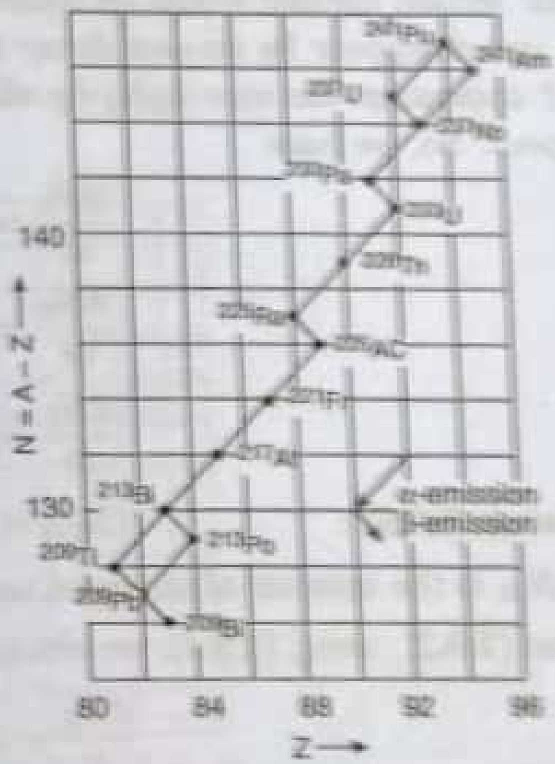

Fig. 2.9 Neptunium series

Neptunium series — After the discovery of *nuclear fission* and the production of *transuranic elements* (i.e., man-made elements beyond uranium, $Z > 92$), a fourth radioactive series called the *neptunium* $(Z = 93)$ *series* has been observed. The series starts with plutonium, ${}^{241}_{94}Pu$ and finishes at ${}^{209}_{83}Bi$, a stable isotope of bismuth. It is also called $(4n+1)$ *series* as the mass number of any nuclide in the family is given by $A = 4n + 1$. Its (N, Z)-chart is shown in Fig. 2.9.

• The several *branching disintegrations* in the above different series are worth-noting. For example, the $^{214}_{83}$Bi in the uranium series becomes $^{210}_{82}$Pb according to the following two alternative transformations :

$$\text{(i)} \quad ^{214}_{83}\text{Bi} \xrightarrow{\beta} {}^{214}_{84}\text{Po} \xrightarrow{\alpha} {}^{210}_{82}\text{Pb}$$

$$\text{(ii)} \quad ^{214}_{83}\text{Bi} \xrightarrow{\alpha} {}^{210}_{81}\text{Tl} \xrightarrow{\beta} {}^{210}_{82}\text{Pb}$$

• The reason of having exactly four radioactive series is due to the fact that the alpha decay reduces the mass number of the nucleus by 4.

2.9 Successive transformation

Consider a radioactive *nuclide* P, symbolized by script 1, to decay into another radioactive nuclide Q (script 2); the latter again decays into a stable end-product R (script 3). For instance,

$$P(1) \xrightarrow{\lambda_1} Q(2) \xrightarrow{\lambda_2} R(3) \ \text{[stable]}.$$

Let λ_1, λ_2 be the decay constants of nuclides 1 and 2 respectively, and N_1, N_2, N_3 be the number of atoms of the three kinds at any instant t. To determine N_1, N_2, N_3.

The second nuclide would be formed at the rate $\lambda_1 N_1$ by the decay of the parent atom and disappear by its own decay at the rate $\lambda_2 N_2$; the atoms of the end-product i.e. R appear at the rate $\lambda_2 N_2$ by the decay of nuclide 2, but being stable do not disappear. So, we have

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \tag{2.9.1}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \tag{2.9.2}$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 \tag{2.9.3}$$

Solving (2.9.1) :

$$(N_1)_t = N_{10} e^{-\lambda_1 t} \tag{2.9.4}$$

where N_{10} is the number of atoms of nuclide 1 at time $t = 0$.

From (2.9.2), using (2.9.4), we obtain

$$\frac{dN_2}{dt} = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2$$

$$\Rightarrow \quad \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{10} e^{-\lambda_1 t} \tag{2.9.5}$$

Multiplying both sides of (2.9.5) by $e^{\lambda_2 t}$, the integrating factor,

$$e^{\lambda_2 t}\left(\frac{dN_2}{dt} + \lambda_2 N_2\right) = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}$$

$$\text{or,} \quad \frac{d}{dt}(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}$$

Integrating, $$N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} e^{(\lambda_2 - \lambda_1)t} + A \tag{2.9.6}$$

where A is the constant of integration.

Now at $t = 0$, $(N_2)_t = N_{20} = 0$, since only the first nuclide is present. So, from (2.9.6), we have

$$A = -\frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}$$

$$\therefore \quad (N_2)_t = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \tag{2.9.7}$$

This equation is known as the ***Batemann equation*** and it gives the number of daughter atoms at time t.

From (2.9.3), using (2.9.7), we obtain, on integration

$$(N_3)_t = \left(\frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\right) e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} N_{10} e^{-\lambda_1 t} + B \tag{2.9.8}$$

At $t = 0$, $(N_3)_t = N_{30} = 0$. So from equation (2.9.8), we get $B = N_{10}$.

$$\therefore \quad (N_3)_t = N_{10}\left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}\right) \tag{2.9.9}$$

Equations (2.9.4), (2.9.7) and (2.9.9) therefore constitute the solution of the given problem.

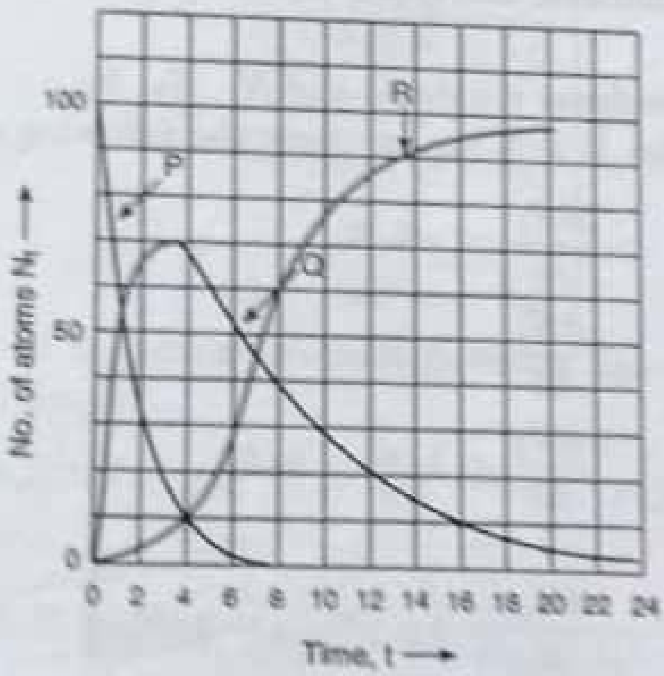

Fig. 2.10 Successive disintegration : Growth of daughter and decay of parent atoms

The decay of the first nuclide P, the growth and decay of the second nuclide Q and the growth of the third nuclide R are shown in Fig. 2.10.

Time for Q(2) to attain a maximum — Eq. (2.9.7) gives the number of daughter atoms N_2 at time t i.e., how N_2 varies with t. It shows that $N_2 = 0$ at $t = 0$.

It increases with increasing t and attains a maximum at $t = t_m$, say. This t_m can be determined by imposing the condition $dN_2/dt = 0$, at $t = t_m$.

$$\text{Now} \quad \frac{dN_2}{dt} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} \left(-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} \right)$$

$$\therefore \quad dN_2/dt = 0 \Rightarrow \lambda_2 e^{-\lambda_2 t_m} = \lambda_1 e^{-\lambda_1 t_m}$$

$$\therefore \quad e^{(\lambda_2 - \lambda_1)t_m} = \frac{\lambda_2}{\lambda_1} \Rightarrow (\lambda_2 - \lambda_1)t_m = \ln \frac{\lambda_2}{\lambda_1}$$

$$\therefore \quad \boxed{t_m = \frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}} \tag{2.9.10}$$

That this t_m corresponds to maximum N_2 can be readily verified by obtaining the second time-derivative of N_2 which becomes negative at $t = t_m$.