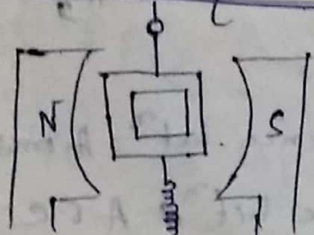
(Subhash ch. Sen) **Eletro magnetic induction** Dept:- Physics (H) Sem = II Paper - C_{3T}

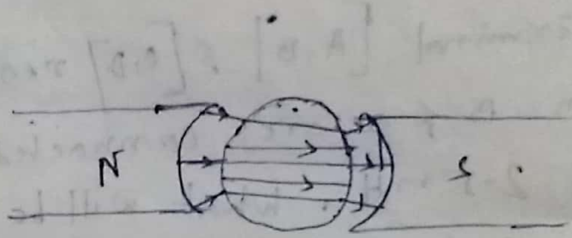
Reciprocity theorem :-

It states that, " In a circuit Containing liner ohmic resistant and energy Source, the response remains the some when the Position of excitation and reponce are inter change.

Ex:- In case of wheatstone Bridge the Position of ammeter and batary can be inter change and in both case ammeter show the same reading

Ballistics Galvanometer :-

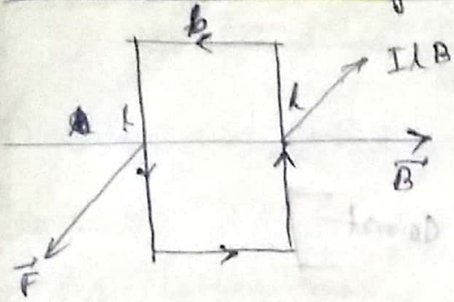




Describtion :-

B.G consists of a coil of large no. of turns. Wound on an insulating frame. The coil is suspended from a torson head by a thin strip of Phosphon bronze fibre and has a narrow space bet^n the radial Pole Pieces of a Parmanent magnet and cylindrical shopt o iron core. The electro magnetic damping is reduce to a minimum by wring and insulating fram. The time Period of G. galvanometer made large arround 10 to 15 sec, so that coil does not move appreciably before the whole of charge Passes through it. This gives an impulse to coil. As the horizontal arm of the coil are always ||el to the magnetic line of force, the force on the arme is zero.

Mathematical theory



If the current is I through the coil of length 'l' and breadth 'b'

∴ Force on the vertical arms is :

$$\vec{F} = I\int \vec{dl} \times \vec{B}$$

$$\therefore F = IlB$$

As the force act for a very short duration so impuls given to the vertical arm is

$$\int_0^{t_0} F\,dt = \int_0^{t_0} IlB\,dt = \int lB\,dq \qquad \left[I = \frac{dq}{dt}\right]$$

$$= lBq$$

For n terms of the coil total impuls is $lBq \times n$

∴ Moment of impuls given to the coil is

$$lBqnb = nABq \qquad \left[\begin{array}{l} A = \text{area of the coil} \\ = lb \end{array}\right]$$

The causes a change in angular momentum of the coil and it begin to rotated with angular velocity ω

$$\therefore nABq = I_m\omega \qquad \left[\begin{array}{l} I_m = \text{Moment of inertia of the coil about the fibre} \end{array}\right]$$

As the coil rotated the fibre is twisted, the inerising electric potential energy occurs at the expenses of the kinetic energy of the coil

$$\therefore \frac{1}{2}C\theta_0^2 = \frac{1}{2}I_m\omega^2 \qquad \left[\begin{array}{l} C = \text{torsonal rigidity of fibre} \\ \theta_0 = \text{Maximum deflection of the coil} \end{array}\right]$$

$$\therefore \omega^2 = \frac{C\theta_0^2}{I_m}$$

Again $nABq = I_m\omega$

$$\therefore n^2A^2B^2q^2 = I_m^2\omega^2$$

$$\Rightarrow n^2A^2B^2q^2 = I_m^2\frac{C\theta_0^2}{I_m} = I_m C\theta_0^2$$

$$q^2 = \frac{I_m C\theta_0^2}{n^2A^2B^2}$$

[Time period of torsonal ossilation of the coil $T = 2\pi\sqrt{I_m/C}$, $T^2 = 4\pi^2\frac{I_m}{C}$

$I_m = \frac{CT^2}{4\pi^2}$]

$$= \frac{CT^2}{4\pi^2}\cdot\frac{C\theta_0^2}{n^2A^2B^2}$$

nAB

$$q = \frac{CT\theta_0}{2\pi nAB}$$

$\therefore$ $q \propto \theta_0$ $\left[\frac{CT}{2\pi nAB} = \text{const}\right]$

(1) There are two types of damping in the B.G

(i) Mechnical damping :-

As the coil move in air, the viscous force of air will give a resistant to the ~~moment~~ movement of coil this is mechnical damping and damping couple acting on the coil is $A\frac{d\theta}{dt}$

(ii) Electro magnetic damping

When the coil moves in the field of permanent magnet, the flux linked with it changes, producing an induced e.m.f, The induced current in the coil produced an anti couple, damping the motion of coil this is electromagnetic damping and let it is $Q\frac{d\theta}{dt}$.

$\therefore$ Total damping couple acting on the coil is

$(A+Q)\frac{d\theta}{dt}$ ~~$= 2b\frac{d\theta}{dt}$~~ ~~$(A+Q = 2B)$~~

$\therefore$ Equation of motion of the coil —

$$I_m \frac{d^2\theta}{dt^2} + (A+Q)\frac{d\theta}{dt} + C\theta = 0$$

$$\therefore \frac{d^2\theta}{dt^2} + 2b\frac{d\theta}{dt} + \omega_0^2\theta = 0 \quad \left[\frac{A+Q}{I_m} = 2b\right]$$

$\left[\frac{C}{I_m} = \omega_0^2\right]$

The solution is

$$\theta = Ae^{-bt\sqrt{b^2-\omega^2}\,t} + Be^{-bt\sqrt{b^2-\omega^2}}$$

Case - 1

critical damping

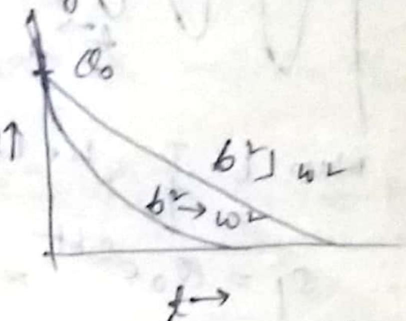
The condition is $b \to \omega$

In that case the coil came to rest very quickly

Case-2

Over damped motion :-

The condition is $b > w$. In that case the motion is periodic and the coil gradually comes to rest.



Case I & 2

Case-3

Oscilatory motion of coil

The condition is $b < w$

$\sqrt{b^2 - w^2} = i\sqrt{w^2 - b^2}$

$\therefore \theta = e^{-bt}\left(Ae^{i\sqrt{w^2-b^2}\,t} + Be^{-i\sqrt{w^2-b^2}\,t}\right)$

$= e^{-bt}\left[A\cos pt + iA\sin pt + B\cos pt - iB\sin pt\right]$ $\left[\sqrt{w^2-b^2} = p\right]$

$= e^{-bt}\left[(A+B)\cos pt + i(A-B)\sin pt\right]$ $\left[\begin{array}{l} A+B = c\cos\delta \\ i(A-B) = c\sin\delta \end{array}\right]$

$= ce^{-bt}\left[\cos\delta\cos pt + \sin\delta\sin pt\right]$

$= ~~ce^{-bt}\cos\delta - pt~~ \; ce^{-bt}\cos(pt - \delta)$

The amplitude of damp oscillation is

~~$c\cos(pt-\delta) = c\cos\left(\sqrt{w^2-b^2}\,t - \delta\right)$~~

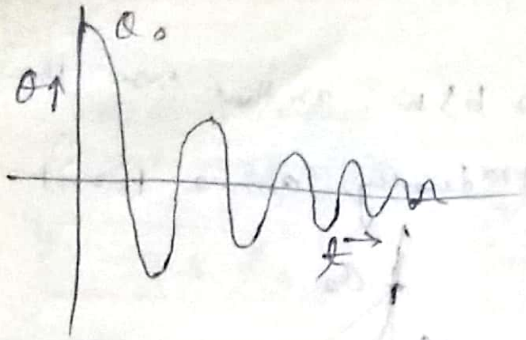
$= ce^{-bt}$

At $t = 0$, $\theta = \theta_0$

$\therefore$ ~~$\theta_0 = c\cos\delta$~~ $\theta_0 = c$

$\therefore \theta = \theta_0 e^{-bt}\cos(pt - \delta)$

We get the gradual decrease of is amplitude of swing the coil



When $t = \frac{T}{4}$ we have $\theta = \theta_1$, the first ~~through~~ throw

$\therefore \theta_1 = \theta_0 e^{-bt} = \theta_0 e^{-\frac{bT}{4}}$

2nd throw when $t = \frac{3}{4}T = \left(T/2 + T/4\right)$

$\therefore \theta_2 = \theta_0 e^{-bt} = \theta_0 e^{-b\frac{3T}{4}}$

$$\therefore \frac{\theta_1}{\theta_2} = \frac{\theta_0 e^{-\frac{bT}{4}}}{\theta_0 e^{-\frac{b3T}{4}}} = e^{-\frac{bT}{4} + \frac{b3T}{4}} = e^{\frac{bT}{2}} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4}$$

$$= e^{\lambda} \quad \left[\lambda = \frac{bT}{2}\right] = \text{logarithmic decrement}$$

Now $\frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \frac{\theta_3}{\theta_4} \cdots \frac{\theta_n}{\theta_{n+1}} = e^{\lambda} \times e^{\lambda} \times e^{\lambda} \cdots$

$$= e^{n\lambda}$$

$$\therefore \frac{\theta_1}{\theta_{n+1}} = e^{n\lambda}$$

$$\log \frac{\theta_1}{\theta_{n+1}} = n\lambda$$

$$\boxed{\therefore \lambda = \frac{1}{n} \log \frac{\theta_1}{\theta_{n+1}}}$$

Sensitivity of galvanometer :-

The deflection of the spot of light in milimeter on a seal placed normally at a distant of 1 meter form the galvanometer

i) current sensitivity

When ~~meter~~ $1\mu A \, (10^{-6} A)$ current passed throw the Galvanometer

$$S_i = \frac{\theta}{I} = \frac{nAB}{c}$$

(ii) voltage sensitivity

⇒

When the potential diffⁿ is 1μv across the galvanometer

$$S_v = \frac{S_i}{r_g}$$ [r_g = galvanometer resistant]

(iii) charge sensitivity

⇒

When 1μ coulmb (1μc) charge passes throw the galvanometer

$$q = \frac{cT\theta_0}{2\pi nAB}$$

∴ S_q = charge sensitivity

$$= \frac{\theta_0}{q} = \frac{2\pi}{T}\frac{nAB}{c} = \frac{2\pi}{T}S_i$$ [$S_i = \frac{nAB}{c}$]

(iv) Figure of merit :-

$$= \frac{1}{\text{current sensitivity}} = \frac{c}{nAB}$$