- SEMESTER –IV (HONOURS)
- PAPER : CC9T
- TOPIC: NUCLEAR SHELL MODEL

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Spin – orbit potential

However, the introduction of this additional force only removes the degeneracy with respect to *l* but will not yield the correct magic numbers above 20. So, Mayer and Jenson independently in 1949 introduced a spin-orbit coupling $l \cdot s$. *Originally, the spin-orbit force was introduced phenomenologically to reproduce the magic numbers but now it is believed to arise from the tensor force of the nucleon-nucleon interaction.*

$$\begin{aligned} \boldsymbol{l}\cdot\boldsymbol{s} &= \frac{1}{2}\left(\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2\right) \\ \langle \boldsymbol{l}\cdot\boldsymbol{s}\rangle &= \frac{1}{2}\{j(j+1) - l(l+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\} \\ &= \begin{cases} \frac{1}{2}l & \text{for } j = l + \frac{1}{2}, \\ -\frac{1}{2}(l+1) & \text{for } j = l - \frac{1}{2}. \end{cases} \end{aligned}$$

Thus we see that the spin–orbit force removes the degeneracy of the level with respect to spin and from (6.22), we find the spacing between the energy levels with $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ to be proportional to $(2l+1)V_{SO}$. Let us assume the spin–orbital potential V_{SO} to be of the Thomas form,

$$V_{SO} = U_{SO}\, \boldsymbol{l}\cdot\boldsymbol{s} = -\frac{1}{r}\frac{\partial V}{\partial r}\, \boldsymbol{l}\cdot\boldsymbol{s}.$$

For the harmonic oscillator potential, $V \sim r^2$ and consequently $(1/r)(\partial V/\partial r)$ is a constant. The negative sign in expression (6.23) lowers the levels with higher j value. By this artifice, we are able to produce bunching of levels and a large spacing between any two bunches. The magic number corresponds to the number of protons or neutrons that occupy the bunch of closely spaced single particle levels separated by large spacing. From Table 6.5 as well as from Fig. 6.1, we find that the magic numbers correspond to fairly large separations of closely spaced single particle states. It is found empirically that the correct level scheme can be obtained from the expression

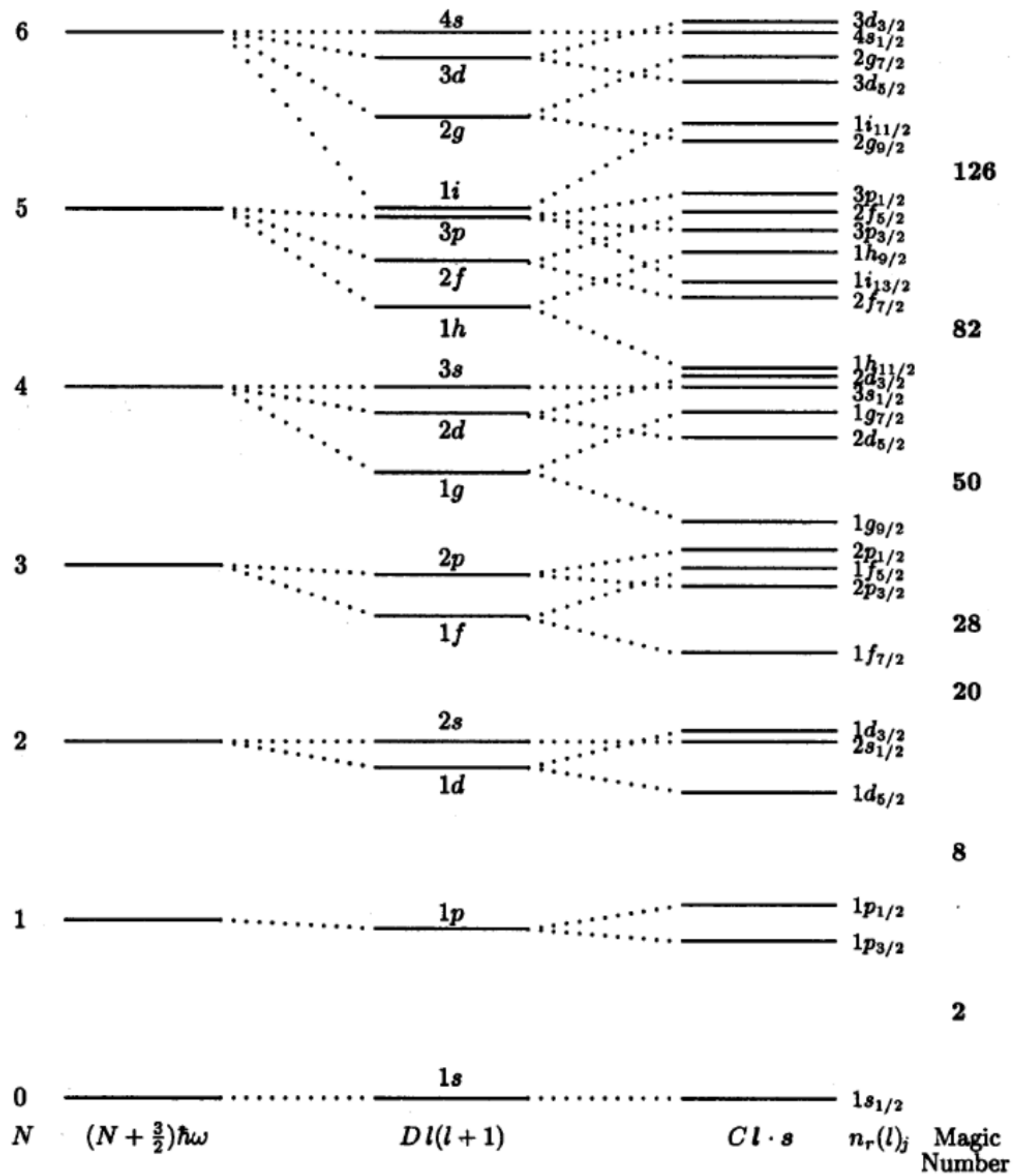
$$E = \left(N + \frac{3}{2}\right)\hbar\omega + Dl(l+1) + C\,\boldsymbol{l}\cdot\boldsymbol{s},$$

with $D = -0.0225\,\hbar\omega$ and $C = -0.1\,\hbar\omega$. We now obtain the magic numbers correctly. Even more important is that we get the correct assignment for the spin and parity of the ground state of the odd-mass nuclei, assuming that the spin of the odd-mass nucleus is equal to the spin of the last nucleon.

Table 6.5: The inclusion of spin-orbit coupling causes the grouping of levels into various shells and explains the magic numbers of protons or neutrons that correspond to the closed shell structures.

Shell	States	No. of protons or neutrons in the shell	Total number of protons or neutrons
I	$1s$	2	2
II	$1p_{3/2}$, $1p_{1/2}$	4, 2	8
III	$1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$	6, 2, 4	20
IV	$1f_{7/2}$	8	28
V	$2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$	4, 6, 2, 10	50
VI	$1g_{7/2}$, $2d_{5/2}$ $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$	8, 6, 4, 2, 12	82

The single-particle energy level scheme with spin-orbit coupling is shown in Fig. I for the nuclear shell model yielding correctly the magic numbers.



6
5
4
3
2
1
0
N
$(N+\frac{3}{2})\hbar\omega$
4s
3d
2g
1i
3p
2f
1h
3s
2d
1g
2p
1f
2s
1d
1p
1s
$D\,l(l+1)$
$3d_{3/2}$
$4s_{1/2}$
$2g_{7/2}$
$3d_{5/2}$
$1i_{11/2}$
$2g_{9/2}$
$3p_{1/2}$
$2f_{5/2}$
$3p_{3/2}$
$1h_{9/2}$
$1i_{13/2}$
$2f_{7/2}$
$1h_{11/2}$
$2d_{3/2}$
$3s_{1/2}$
$1g_{7/2}$
$2d_{5/2}$
$1g_{9/2}$
$2p_{1/2}$
$1f_{5/2}$
$2p_{3/2}$
$1f_{7/2}$
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
$C\,l\cdot s$
$n_r(l)_j$
126
82
50
28
20
8
2
Magic Number

Predictions of the shell model:

1. Stability of the close shell nuclei. This scheme clearly reproduces all the magic numbers.

2. Spin and Parity of Nuclear Ground States.

   the shell model has been very successful predicting the ground state spin of a large number of nuclei.

   i. Even-even nuclides (both Z and A even) have zero intrinsic spin and even parity.
   ii. Odd A nuclei have one unpaired nucleon. The spin of the nucleus is equal to the jvalue of that unpaired nucleon and the parity is $(-1)^l$, where l is the orbitalangular momentum of the unpaired nucleon.

   iii. Odd-odd nuclei. In this case there is an unpaired proton whose total angular momentum is j_1 and an unpaired neutron whose total angular momentum is j_2. The total
   spin of the nucleus is the (vector) sum of these angular momenta and can take values between $|j_1 - j_2|$ and $|j_1 + j_2|$ (in unit steps). The parity is given by $(-1)^{(L_1+L_2)}$, where l_1 and l_2 are the orbital angular momenta of the unpaired proton and neutron respectively.

   For example $_3Li^6$(lithium) has 3 neutrons and 3 protons. The first two of each fill the 1s level and the thrid is in the $1p_{3/2}$ level. The orbital angular mometum of each is l = 1
   so the parity is (-1)×(-1) = +1 (even), but the spin can be anywhere between 0 and 3.

IV. Magnetic Dipole Moments

Since nuclei with an odd number of protons and/or neutrons have intrinsic spin they also in general possess a magnetic dipole moment.

The unit of magnetic dipole moment for a nucleus is the "nuclear magneton" defined as

$$\mu_N \ = \ \frac{e\hbar}{2m_p},$$

which is analogous to the Bohr magneton but with the electron mass replaced by the proton mass. It is defined such that the magnetic moment due to a proton with orbital angular momentum $\mathbf{l}$ is $\mu_N \, \mathbf{l}$.

Experimentally it is found that the magnetic moment of the proton (due to its spin) is

$$\mu_p \ = \ 2.79\mu_N \ = \ 5.58\mu_N s, \quad \left(s = \frac{1}{2}\right)$$

and that of the neutron is

$$\mu_n \ = \ -1.91\mu_N \ = \ -3.82\mu_N s, \quad \left(s = \frac{1}{2}\right)$$

If we apply a magnetic field in the z-direction to a nucleus then the unpaired proton with orbital angular momentum $\mathbf{l}$, spin $\mathbf{s}$ and total angular momentum $\mathbf{j}$ will give a contribution to the $z-$ component of the magnetic moment

$$\mu^z \ = \ (5.58 s^z \ + \ l^z)\, \mu_N.$$

As in the case of the Zeeman effect, the vector model may be used to express this as

$$\mu^z \ = \ \frac{(5.58 < \mathbf{s} \cdot \mathbf{j} > + < \mathbf{l} \cdot \mathbf{j} >)}{< \mathbf{j}^2 >} j^z \, \mu_N$$

using

$$\begin{aligned}
< \mathbf{j}^2 > \ &= \ j(j+1)\hbar^2 \\
< \mathbf{s} \cdot \mathbf{j} > \ &= \ \frac{1}{2} \left(< \mathbf{j}^2 > + < \mathbf{s}^2 > - < \mathbf{l}^2 >\right) \\
&= \ \frac{\hbar^2}{2} \left(j(j+1) \ + \ s(s+1) \ - \ l(l+1)\right) \\
< \mathbf{l} \cdot \mathbf{j} > \ &= \ \frac{1}{2} \left(< \mathbf{j}^2 > + < \mathbf{l}^2 > - < \mathbf{s}^2 >\right) \\
&= \ \frac{\hbar^2}{2} \left(j(j+1) \ + \ l(l+1) \ - \ s(s+1)\right) \qquad (5.4.1)
\end{aligned}$$

We end up with expression for the contribution to the magnetic moment

$$\mu \ = \ \frac{5.58 \left(j(j+1) \ + \ s(s+1) \ - \ l(l+1)\right) + \left(j(j+1) \ + \ l(l+1) \ - \ s(s+1)\right)}{2j(j+1)} j \, \mu_N$$

and for a neutron with orbital angular momentum l' and total angular momentum j' we get (not contribution from the orbital angular momentum because the neutron is uncharged)

$$\mu = -\frac{3.82\left(j'(j'+1) + s(s+1) - l'(l'+1)\right)}{2j'(j'+1)} j' \mu_N$$

Thus, for example if we consider the nuclide $^7_3$Li for which there is an unpaired proton in the $2p_{\frac{3}{2}}$ state ($l = 1,\ j = \frac{3}{2}$ then the estimate of the magnetic moment is

$$\mu = \frac{5.58\left(\frac{3}{2}\times\frac{5}{2} + \frac{1}{2}\times\frac{3}{2} - 1\times 2\right) + \left(\frac{3}{2}\times\frac{5}{2} + 1\times 2 - \frac{1}{2}\times\frac{3}{2}\right)}{2\times\frac{3}{2}\times\frac{5}{2}} \frac{3}{2} = 3.79\mu_N$$

The measured value is $3.26\mu_N$ so the estimate is not too good. For heavier nuclei the estimate from the shell model gets much worse.

The precise origin of the magnetic dipole moment is not understood, but in general they cannot be predicted from the shell model. For example for the nuclide $^{17}_{9}$F (fluorine), the measured value of the magnetic moment is $4.72\mu_N$ whereas the value predicted form the above model is $-0.26\mu_N$. !! There are contributions to the magnetic moments from the nuclear potential that is not well-understood.

Disadvantage :

1. There exists difference between shell model wave function and the real state of the nucleus.
2. The large value of Q the quadrupole moment in many nuclei can not be explained with this mode.
3. The strong spin – orbit interaction , are certainly not applicable in this model.
4. The shell model can not be applied to many heavy nuclei.

References :

1. Nuclear physics book by D.C TAYAL
2.