complexities.

8.2 WAVE MOTION :

Any physical entity which varies both in space and time is said to constitute a *wave*. The most common example of a wave is that produced on the surface of water by dropping a stone on it. Here the displacement of the water particles is a function of both space and time. If we take a snapshot of the wave at any instant of time we get the variation of displacement with space coordinates. Again, if we look at a particle

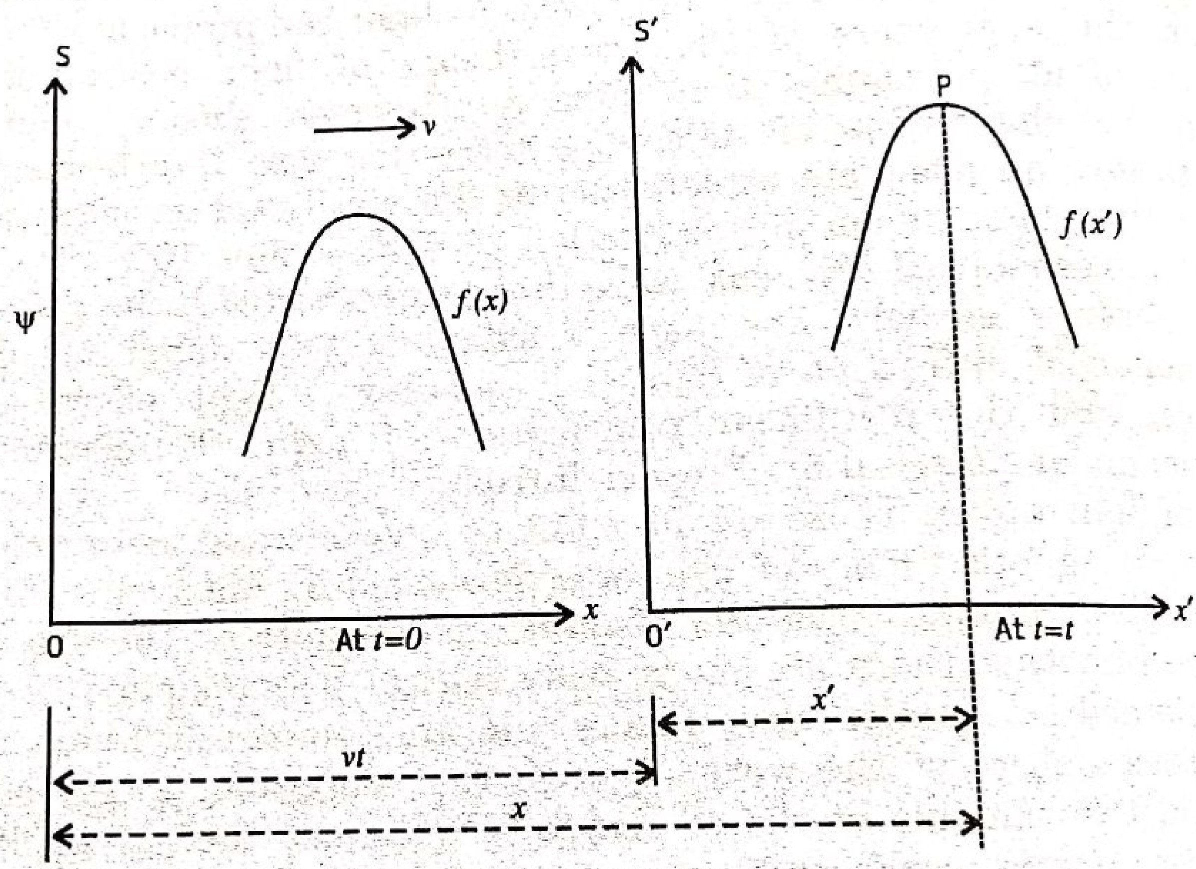

Fig. 8.2-1

at one particular space point we get the variation of displacement with time. If these variations are periodic we speak of *periodic waves*. Mathematical theory of wave motion begins with periodic waves which do not change its shape as it travels through a medium. Such a wave is called *periodic wave of constant type*.

Let the 'disturbance' be a function of space coordinate x and time t and be represented as

$$\psi = f(x, t) \quad \ldots(8.2\text{-}1)$$

Here we are considering one dimensional wave. The shape of the disturbance or *wave profile* at any instant, say $t = 0$, is given by

$$\psi(x, t)\big|_{t=0} = f(x, 0) = f(x) \quad \ldots(8.2\text{-}2)$$

Now assume that the wave configuration moves along positive x-direction with a velocity v. So in time t it moves through a distance vt, but in all other respect the wave profile remains unchanged (Fig. 8.2-1). Let us now introduce a coordinate system S' moving with the wave at the speed v. At S-time $t = 0$, let the origins O and O' coincide.

With reference to S' frame the wave profile appears to be stationary with the same functional form as Eq. (8.2-2). Thus in S' frame

$$\psi = f(x') \qquad \text{...(8.2-3)}$$

Now at any time t, the coordinates of any point P on the wave profile with respect to S and S' are related by

$$x' = x - vt \qquad \text{...(8.2-4)}$$

Thus with reference to stationary S-frame we can write

$$\psi(x, t) = f(x - vt) \qquad \text{...(8.2-5)}$$

This represents the most general form of one dimensional wave propagating along the positive x-direction.

Similarly, a wave propagating along negative x-direction may be represented as

$$\psi(x, t) = f(x + vt) \qquad \text{...(8.2-6)}$$

Differential wave equation :

From the arbitrary wave function $\psi(x, t) = f(x \mp vt)$ we can derive the one dimensional differential wave equation. Writing $x' = x \mp vt$ we get $\psi(x, t) = f(x')$

$$\therefore \quad \frac{\partial \psi}{\partial x} = \frac{df}{dx'} \cdot \frac{\partial x'}{\partial x} = \frac{df}{dx'}$$

and
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{d^2 f}{dx'^2} \cdot \frac{\partial x'}{\partial x} = \frac{d^2 f}{dx'^2} \qquad \text{...(8.2-7)}$$

Similarly,
$$\frac{\partial \psi}{\partial t} = \frac{df}{dx'} \cdot \frac{\partial x'}{\partial t} = \frac{df}{dx'} \cdot (\mp v)$$

and
$$\frac{\partial^2 \psi}{\partial t^2} = \frac{d^2 f}{dx'^2} \cdot (\mp v) \cdot \frac{\partial x'}{\partial t} = v^2 \cdot \frac{d^2 f}{dx'^2} \qquad \text{...(8.2-8)}$$

Combining Eqs. (8.2-7) and (8.2-8) we obtain

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \qquad \text{...(8.2-9)}$$

which is the one dimensional **differential wave equation**. It is a linear homogeneous differential equation. Hence the **principle of**

superposition is applicable here. If ψ_1 and ψ_2 are two different solutions then a linear combination of ψ_1 and ψ_2 i.e., $\psi = c_1\psi_1 + c_2\psi_2$ is also a solution. Accordingly most general solution of Eq. (8.2-9) has the following form :

$$\psi = c_1 f_1 (x - vt) + c_2 f_2 (x + vt) \quad \text{...(8.2-10)}$$

where c_1 and c_2 are constants and the functions are twice differentiable. The solution represents two waves of constant profile moving in opposite directions with the same velocity v. However, the waves need not have the same wave profile.

Harmonic waves :

The simplest kind of vibration that a particle of the medium can execute is the simple harmonic vibration. In such a case the associated wave profile is a sine or cosine curve. This is known as *sinusoidal wave* or *harmonic wave*. Such a wave propagating along positive x-direction can be represented as

$$\psi(x, t) = a \sin k(x - vt)$$

or,

$$\psi(x, t) = a \cos k(x - vt) \quad \text{...(8.2-11)}$$

Here a represents maximum disturbance and is known as the *amplitude* of the wave. The wave is periodic both in space and time.

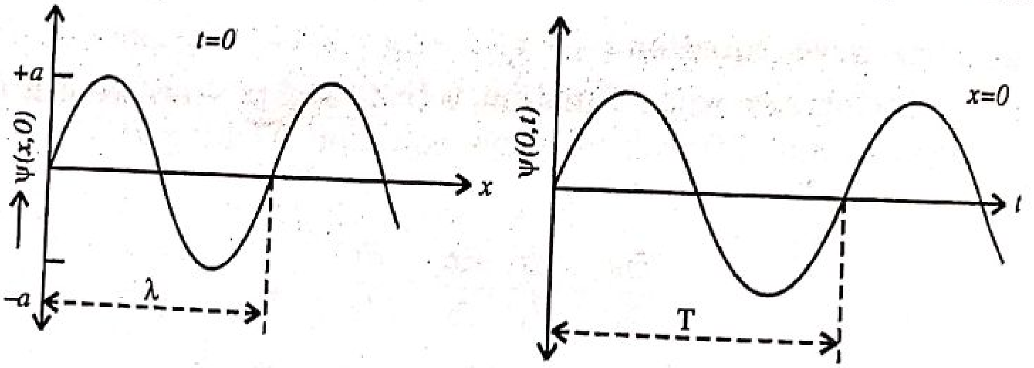

Fig.8.2-2 A harmonic wave

The spatial period is known as *wavelength* and is denoted by λ (Fig. 8.2-2). It is the closest distance between two points having same phase. Now the spatial periodicity requires that

$$\psi(x, t) = \psi(x + \lambda, t)$$

This requires that $k.\lambda = 2\pi$ or, $k = \dfrac{2\pi}{\lambda}$. The constant k is known as **propagation constant**. Similarly, the **temporal period** (T) is given by

$$\psi(x, t) = \psi(x, t + T)$$

This requires that $kv.T = 2\pi$ or, $kv = \dfrac{2\pi}{T} = \omega$ which is the **angular frequency** of the harmonic motion. Eqs. (8.2-11) may also be expressed in the following most frequently used form as

$$\psi(x, t) = a \sin(kx - \omega t) \text{ or, } \psi(x, t) = a \cos(kx - \omega t)$$
$$= Im\ ae^{i(kx - \omega t)} \qquad = Re\ ae^{i(kx - \omega t)} \quad ...(8.2\text{-}12)$$

The entire argument $kx - \omega t$ is known as the **phase** of the wave. It represents the state motion of any particle at any position x and at any time t. Let $\varphi(x, t) = kx - \omega t$. Suppose at any later instant $t + dt$ the same phase occur at point $x + dx$. Then,

$$\varphi(x, t) = \varphi(x + dx,\ t + dt)$$

or,
$$kx - \omega t = k(x + dx) - \omega(t + dt)$$

or,
$$kdx - \omega dt = 0$$

or,
$$\frac{dx}{dt} = \frac{\omega}{k} = v$$

Thus v represents the velocity of propagation of the condition of constant phase. It also represents the velocity of propagation of an unchanging wave form. It is called the **phase** (or **wave**) **velocity**.

Wavefront :

A wavefront is a surface upon which the phase of disturbance is the same at any given instant of time. It is the loci of points of constant phase. A wave travels in a direction normal to the wavefront. In case of a point source in an isotropic medium the wavefronts are spherical surfaces. Such waves are called *spherical waves*. For linear sources wavefronts are cylindrical and corresponding waves are called *cylindrical waves*. When the loci of points of constant phase lie on parallel planes the wavefronts are called plane and the associated waves are called *plane waves*. At a sufficient distance from a source a small portion of the wavefront may be taken as plane. In many optical devices we produce light resembling plane waves. Thus the study of such waves is important.